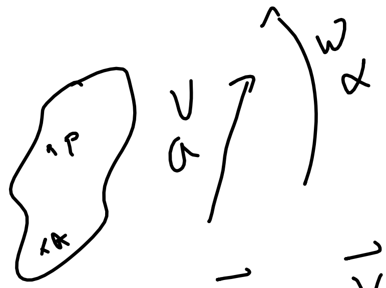
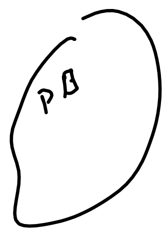
$\cancel{\tau = I\alpha}$



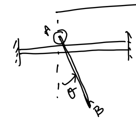
$$\vec{V}_P = \vec{V}_A + \vec{\omega} \times \vec{r}_{P/A}$$

$$\vec{a}_P = \vec{a}_A + \vec{\alpha} \times \vec{r}_{P/A} - \omega^2 \vec{r}_{P/A}$$



1) Draw FBD
2) Choose a point 'p' to compute torque/moment about
3) Compute $\vec{a}_G$
4) $\vec{F} = m\vec{a}_G$ (2-eqns for planar case)

$\vec{M}_P = \sum \vec{r}_{i/P} \times \vec{F}_i = I\vec{\alpha}_G + \vec{r}_{G/P} \times m\vec{a}_G$ (1-eqn for planar)



$I_G = \frac{mL^2}{12}$

$a_A = ?$ $\alpha_{AB} = ?$ Right after release.

$\vec{a}_G = \vec{a}_A + \vec{a}_{G/A}$

$= \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{G/A} - \omega_{AB}^2 \vec{r}_{G/A}$

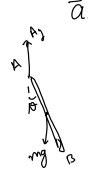
$= a_{A_x} \hat{i}$
$+ \alpha_{AB}\hat{k} \times \left(\frac{L}{2}\sin\theta\,\hat{i} - \frac{L}{2}\cos\theta\,\hat{j}\right)$
$- \omega_{AB}^2 \left(\frac{L}{2}\sin\theta\,\hat{i} - \frac{L}{2}\cos\theta\,\hat{j}\right)$

$\vec{a}_G = a_x\hat{i} + \alpha_{AB}\frac{L}{2}\sin\theta\,\hat{j} + \frac{\alpha_{AB}L}{2}\cos\theta\,\hat{i}$
$- \omega_{AB}^2\frac{L}{2}\sin\theta\,\hat{i} + \frac{\omega_{AB}^2 L}{2}\cos\theta\,\hat{j}$

$= \left(a_x + \frac{\alpha_{AB}L}{2}\cos\theta - \frac{\omega_{AB}^2}{2}L\sin\theta\right)\hat{i}$
$+ \left(\frac{\alpha_{AB}L}{2}\sin\theta + \frac{\omega_{AB}^2 L}{2}\cos\theta\right)\hat{j}$

As AB is released from rest, $\omega_{AB} = 0$.

$\vec{a}_G = \left(a_x + \frac{\alpha_{AB}L}{2}\cos\theta\right)\hat{i} + \frac{\alpha_{AB}L\sin\theta}{2}\hat{j}$



$\sum \vec{F} = m\vec{a}_G$ in x & y dxn

$\vec{M}_A = I_G\vec{\alpha}_{AB} + \vec{r}_{G/A} \times m\vec{a}_G$ ✓

$\vec{M}_G = I_G\vec{\alpha}_{AB}$ ✓

$\vec{M}_P = \sum_{i=1}^{n} \vec{r}_{i/P} \times \vec{F}_i$

$\underline{x}$ $\sum F_x = ma_x$

$\Rightarrow$ $0 = m\left(a_x + \frac{\alpha_{AB}L}{2}\cos\theta\right)$ — ①

$\underline{y}$ $\sum F_y = ma_y$

$A_y - mg = m\left(\frac{\alpha_{AB}L}{2}\sin\theta\right)$ — ②

$\hat{k}$// $\vec{M}_A = \sum \vec{r}_{i/A} \times \vec{F}_i$
$= \left(\frac{L}{2}\sin\theta\,\hat{i} - \frac{L}{2}\cos\theta\,\hat{j}\right) \times (-mg\,\hat{j})$
$= -\frac{mgL}{2}\sin\theta\,\hat{k}$

$I_G\vec{\alpha}_G + \vec{r}_{G/A} \times m\vec{a}_G$

$= \frac{mL^2}{12}\alpha_{AB}\hat{k} + \left[\left(\frac{L}{2}\sin\theta\,\hat{i} - \frac{L}{2}\cos\theta\,\hat{j}\right) \times m\left[\left(a_x + \frac{\alpha_{AB}L}{2}\cos\theta\right)\hat{i} + \frac{\alpha_{AB}L}{2}\sin\theta\,\hat{j}\right]\right]$

$= \frac{mL^2}{12}\alpha_{AB}\hat{k} + m\left[\frac{L^2}{4}\alpha_{AB}\sin^2\theta\,\hat{k} + \frac{a_x L}{2}\cos\theta\,\hat{k} + \frac{L^2\alpha_{AB}}{4}\cos^2\theta\,\hat{k}\right]$

$= \left[\frac{mL^2\alpha_{AB}}{12} + \frac{mL^2\alpha_{AB}}{4} + \frac{ma_x L}{2}\cos\theta\right]\hat{k}$

$= \left(\frac{mL^2\alpha_{AB}}{3} + \frac{ma_x L}{2}\cos\theta\right)\hat{k}$

$$-\frac{mgL}{2}\sin\theta = \frac{mL^2}{3}\alpha_{AB} + \frac{ma_xL}{2}\cos\theta$$

$$A_y - mg = \frac{m\alpha_{AB}L}{2}\sin\theta$$

$$0 = m\left(a_x + \frac{\alpha_{AB}L}{2}\cos\theta\right)$$

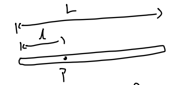
$$a_x = -\frac{\alpha_{AB}L}{2}\cos\theta$$

$$-\frac{\cancel{m}g\cancel{L}}{2}\sin\theta = \frac{\cancel{m}L^{\cancel{2}}\alpha_{AB}}{3} + \frac{\cancel{m}\cancel{L}}{\cancel{2}}\cos\theta \times \left(-\frac{\alpha_{AB}L}{2}\cos\theta\right)$$

$$-6g\sin\theta = 4L\alpha_{AB} - 3L\alpha_{AB}\cos^2\theta$$

$$\alpha_{AB} = \frac{-6g\sin\theta}{L(4-3\cos^2\theta)} \quad //$$

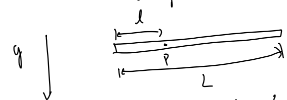
$$a_x = \frac{6g\sin\theta\cos\theta}{2(4-3\cos^2\theta)} \quad //$$



m, L

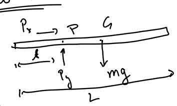
release from rest.

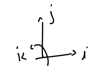
compute 'l' so α_{bar} is maximum.



l so that α_{Bar} is max after releasing from rest.

1) Draw FBD





2) $\bar{a}_G$, α_{bar}, ω_{bar}

$$\vec{a}_G = \cancel{\vec{a}_P}^{\,0} + \vec{a}_{G/P}$$

$$= \vec{\alpha}_{bar} \times \vec{r}_{G/P} - \cancel{\omega^2_{bar}}^{\,0}\, \vec{r}_{G/P}$$

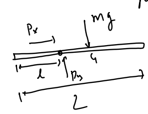
$$= \alpha_{bar}\, \hat{k} \times \left(\frac{L}{2} - l\right) \hat{i}$$

$$= \alpha_{bar} \left(\frac{L}{2} - l\right) \hat{j}$$

3) Choose a point 'p' about which to compute torques.

$$\vec{M}_P = I_G \vec{\alpha}_{bar} + \vec{r}_{G/P} \times m \vec{a}_G$$

if $P \equiv G$, $\vec{M}_G = I_G \vec{\alpha}_{bar}$



4) Write Newton-Euler eqns

$\Sigma F_{x,y} = m a_{x,y}$ ✓

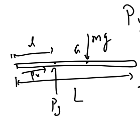
$\vec{M}_G = I_G \alpha_{bar}$ ✓

$\vec{a}_G = \alpha_{bar} \left(\frac{L}{2} - l\right) j$ $\qquad I = \frac{ml^2}{12}$

$P_x = m a_x = 0 \quad$ — ①

$P_y - mg = m a_y$

$P_y - mg = m \alpha_{bar} \left(\frac{L}{2} - l\right) \quad$ — ②



$\vec{M}_G = ?$

$M_{mg} = M_{P_x} = 0$

$\vec{P}_y = P_y \hat{j}$

$\vec{r}_{P/G} = -\left(\frac{L}{2} - l\right) \hat{i}$

$\vec{M}_{P_y} = \vec{r}_{P/G} \times \vec{P}_y = -\left(\frac{L}{2} - l\right) P_y \hat{k}$

$-\left(\frac{L}{2} - l\right) P_y = \frac{mL^2}{12} \alpha_{bar} \quad$ — ③

$P_y - mg = m\alpha_{bar} \left(\frac{L}{2} - l\right)$

$\Rightarrow P_y = mg + m\alpha_{bar} \left(\frac{L}{2} - l\right) \quad$ — ②

$3 \times 2 \times 2$

$-\left(\frac{L}{2} - l\right)\left(\cancel{m}g + \cancel{m}\, \alpha_{bar} \left(\frac{L}{2} - l\right)\right] = \frac{\cancel{m}L^2}{12} \alpha_{bar}$

$\times 12 \quad \Rightarrow -3(L - 2l)(2g + \alpha_{bar}(L - 2l)] = L^2 \alpha_{bar}$

$\Rightarrow -6(L - 2l)g = \left(L^2 + 3(L - 2l)^2\right) \alpha_{bar}$

$\Rightarrow -6(L - 2l)g = \left(L^2 + 3L^2 + 12l^2 - 12lL\right) \alpha_{bar}$

$\Rightarrow -6(L - 2l)g = \left(4L^2 + 12l^2 - 12lL\right) \alpha_{bar}$

$$\Rightarrow \alpha_{bar} = \frac{3(2l - L)g}{2(L^2 - 3Ll + 3l^2)}$$

$$\alpha_{bar} = \frac{3g(2l - L)}{2(L^2 - 3lL + 3l^2)}$$

$$\frac{d\alpha_{bar}}{dl} = 0$$

$$y = \frac{f(x)}{g(x)}, \quad \frac{dy}{dx}$$

$$y = f(x)g(x)$$

$$\frac{dy}{dx} = f'(x)g(x) + g'(x)f(x)$$

$$y = \frac{f(x)}{g(x)}$$

$$\frac{dy}{dx} = \frac{f'(x)}{g(x)} - \frac{1}{g(x)^2}g'(x)f(x)$$

$$\frac{dy}{dx} = \frac{g(x)f'(x) - g'(x)f(x)}{g(x)^2}$$

$$\alpha_{bar} = \frac{3(2l - L)}{2(L^2 - 3Ll + 3l^2)}$$

$$\frac{d\alpha_{bar}}{dl} = \frac{2(L^2 - 3Ll + 3l^2)(6) - (-6L + 12l)3(2l-L)}{(\quad)^2} = 0$$

$$\Rightarrow 2(L^2 - 3Ll + 3l^2)\cancel{6} + \cancel{6}(L - 2l)3(2l - L) = 0$$

$$\Rightarrow \begin{array}{l} 2L^2 - 6Ll + 6l^2 \\ -3L^2 + 12lL - 12l^2 \end{array} = 0$$

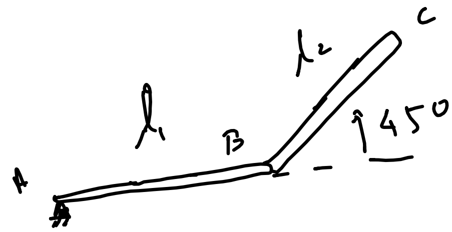
$$\Rightarrow -L^2 + 6Ll - 6l^2 = 0$$

$$\Rightarrow 6l^2 - 6Ll + L^2 = 0$$

$$l = \frac{6 \pm \sqrt{36 - 4 \times 6}}{2 \times 6} L$$

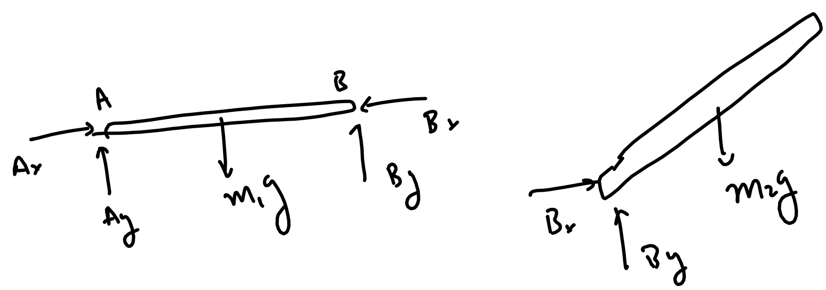
$$= \frac{3 \pm \sqrt{3}}{6} L$$

$$\alpha_{bar} = \pm\frac{\sqrt{3}}{L}g$$



m_1, m_2

$\alpha_{BC}, \alpha_{AB}, A_x, A_y$ 4 unknowns

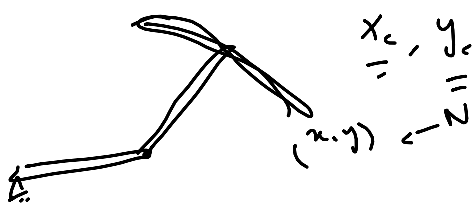


$A_x, A_y, B_x, B_y, \alpha_{AB}, \alpha_{BC}$

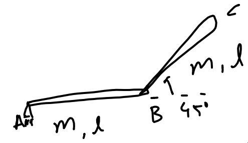
$\Sigma F_x = \ldots$

$\Sigma F_y = \ldots$ // both bodies

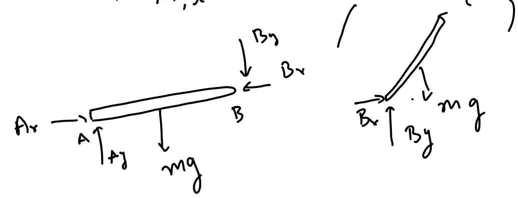
$\Sigma M = \ldots$



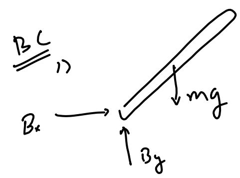
$\underset{=}{x_c}, \underset{=}{y_c}, \underset{=}{R}$ //



α_{AB}, α_{BC}
A_x, A_y
B_x, B_y

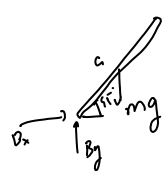


BC 1)



2) $\vec{a}_{G,BC}$

$$\vec{a}_{G,BC} = \vec{a}_B + \vec{a}_{G,BC/B}$$
$$= \vec{a}_A + \vec{a}_{B/A} + \vec{a}_{G,BC/B}$$
$$= \vec{0} + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \cancel{\omega_{AB}^2 \vec{r}_{B/A}}^{\,0}$$
$$+ \vec{\alpha}_{BC} \times \vec{r}_{G,BC/B} - \cancel{\omega_{BC}^2 \vec{r}_{G,BC/B}}^{\,0}$$
$$= \alpha_{AB}\hat{k} \times (l\,\hat{i}) + \alpha_{BC}\hat{k} \times \left(\frac{l}{2}\cos 45\,\hat{i} + \frac{l}{2}\sin 45\,\hat{j}\right)$$
$$\vec{a}_{G,BC} = -\frac{\alpha_{BC}\, l}{2\sqrt{2}}\,\hat{i} + \left(\alpha_{AB}\, l + \alpha_{BC}\frac{l}{2\sqrt{2}}\right)\hat{j}$$

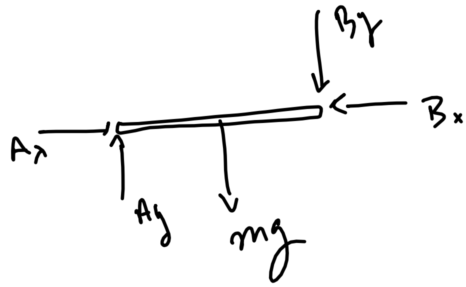


$$\Sigma F_x = m\, a_{G,BCx}$$
$$B_x = m\left(-\frac{\alpha_{BC}\, l}{2\sqrt{2}}\right) \quad -① $$

$$\Sigma F_y = m\, a_{G,BCy}$$
$$\Rightarrow \quad B_y - mg = m\left(\alpha_{AB}\, l + \frac{\alpha_{BC}\, l}{2\sqrt{2}}\right) \quad -②$$

$$\vec{M}_{G,BC} = mg(0)\,\hat{k}$$
$$+ \left(-\frac{l}{2\sqrt{2}}\,\hat{i} - \frac{l}{2\sqrt{2}}\,\hat{j}\right) \times B_x\,\hat{i}$$
$$+ \left(-\frac{l}{2\sqrt{2}}\,\hat{i} - \frac{l}{2\sqrt{2}}\,\hat{j}\right) \times B_y\,\hat{j}$$
$$= \frac{B_x l}{2\sqrt{2}}\,\hat{k} - \frac{B_y l}{2\sqrt{2}}\,\hat{k}$$
$$= \frac{l}{2\sqrt{2}}\left(B_x - B_y\right)\hat{k}$$

$$\vec{M}_{G,BC} = I_G\, \vec{\alpha}_G$$
$$\frac{l}{2\sqrt{2}}\left(B_x - B_y\right) = \frac{ml^2}{12}\,\alpha_{BC} \quad -③$$

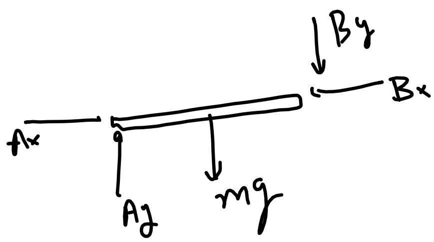


$$\vec{a}_{G,AB} = \cancel{\vec{a}_A}^{\,0} + \vec{a}_{G_{AB}/A}$$

$$= \vec{\alpha}_{AB} \times \vec{r}_{G_{AB}/A} - \cancel{\omega_{AB}^2 \vec{r}_{G,AB/A}}^{\,0}$$

$$= \alpha_{AB}\,\hat{k} \times \left(\frac{l}{2}\,\hat{i}\right)$$

$$\vec{a}_{G,AB} = \frac{\alpha_{AB} l}{2}\,\hat{j}$$



$$\sum F_x = m\,a_{xG,AB}$$

$$A_x - B_x = 0 \quad -④$$

$$\sum F_y = m\,a_{yG,AB}$$

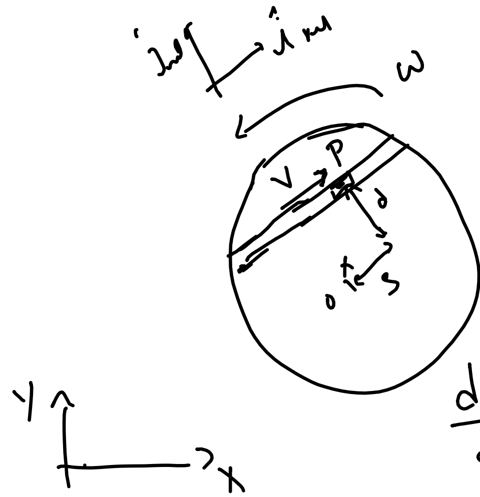
$$\Rightarrow \quad A_y - B_y - mg = \frac{m\alpha_{AB} l}{2} \quad -⑤$$

$$\sum \vec{M}_{G,AB} = I_G \vec{\alpha}_G$$

$$-\frac{l}{2}\,\hat{i} \times A_y\,\hat{j} + \frac{l}{2}\,\hat{i} \times (-B_y\,\hat{j}) = \frac{ml^2}{12}\,\alpha_{AB}\,\hat{k}$$

$$-\frac{l}{2}\left(A_y + B_y\right) = \frac{ml^2}{12}\,\alpha_{AB} \quad // \quad -⑥$$

May 4, 2017



$$\vec{V}_{P/O} = ?$$

$$\vec{r}_{P/O} = s\,\hat{i}_{rel} + d\,\hat{j}_{rel}$$

$$\frac{d\vec{r}_{P/O}}{dt} = \vec{V}_{P/O} = \underbrace{\dot{s}}_{=V}\,\hat{i}_{rel} + \cancelto{0}{\dot{d}}\,\hat{j}_{rel}$$

$$+ s\,\dot{\hat{i}}_{rel} + d\,\dot{\hat{j}}_{rel}$$

$$= V\,\hat{i}_{rel} + s\,\bar{\omega}\times\hat{i}_{rel} + d\,\bar{\omega}\times\hat{j}_{rel}$$

$$= V\,\hat{i}_{rel} + \bar{\omega}\times(s\,\hat{i}_{rel} + d\,\hat{j}_{rel})$$

$$= V\,\hat{i}_{rel} + \vec{\omega}\times(s\,\hat{i}_{rel} + d\,\hat{j}_{rel})$$