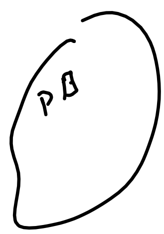
$\tau = I\alpha$

$$\vec{V}_P = \vec{V}_A + \vec{\omega} \times \vec{r}_{P/A}$$



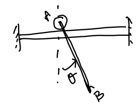
$$\vec{a}_P = \vec{a}_A + \vec{\alpha} \times \vec{r}_{P/A} - \omega^2 \vec{r}_{P/A}$$

May 2, 2017

1) Draw FBD
2) Choose a point 'P' to compute torque/moment about
3) Compute $\vec{a}_G$
4) $\vec{F} = m\vec{a}_G$ (2-eqns for planar case)

$\vec{M}_P = \sum \vec{r}_{i/P} \times \vec{F}_i = I\vec{\alpha}_G + \vec{r}_{G/P} \times m\vec{a}_G$

(1-eqn for planar)



$I_G = \dfrac{mL^2}{12}$

$a_A = ?$ $\alpha_{AB} = ?$ Right after release

$\vec{a}_G = \vec{a}_A + \vec{a}_{G/A}$

$= \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{G/A} - \omega_{AB}^2 \vec{r}_{G/A}$

$= a_{Ax}\hat{i}$

$+ \alpha_{AB}\hat{k} \times \left(\frac{L}{2}\sin\theta\,\hat{i} - \frac{L}{2}\cos\theta\,\hat{j}\right)$

$- \omega_{AB}^2 \left(\frac{L}{2}\sin\theta\,\hat{i} - \frac{L}{2}\cos\theta\,\hat{j}\right)$

$\vec{a}_G = a_x\hat{i} + \alpha_{AB}\frac{L}{2}\sin\theta\,\hat{j} + \frac{\alpha_{AB}L}{2}\cos\theta\,\hat{i}$

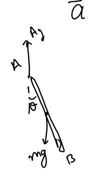
$- \omega_{AB}^2\frac{L}{2}\sin\theta\,\hat{i} + \frac{\omega_{AB}^2 L}{2}\cos\theta\,\hat{j}$

$= \left(a_x + \frac{\alpha_{AB}L}{2}\cos\theta - \frac{\omega_{AB}^2}{2}L\sin\theta\right)\hat{i}$

$+ \left(\frac{\alpha_{AB}L}{2}\sin\theta + \frac{\omega_{AB}^2 L}{2}\cos\theta\right)\hat{j}$

As AB is released from rest, $\omega_{AB} = 0$.

$\vec{a}_G = \left(a_x + \frac{\alpha_{AB}L}{2}\cos\theta\right)\hat{i} + \frac{\alpha_{AB}L\sin\theta}{2}\hat{j}$



$\sum\vec{F} = m\vec{a}_G$ in x & y dxn

$\vec{M}_A = I_G\vec{\alpha}_{AB} + \vec{r}_{G/A} \times m\vec{a}_G$ ✓

$\vec{M}_G = I_G\vec{\alpha}_{AB}$

$\vec{M}_P = \sum_{i=1}^{n} \vec{r}_{i/P} \times \vec{F}_i$ ✓

x: $\sum F_x = ma_x$

$\Rightarrow$ $0 = m\left(a_x + \frac{\alpha_{AB}L}{2}\cos\theta\right)$ — ①

y: $\sum F_y = ma_y$

$A_y - mg = m\left(\frac{\alpha_{AB}L}{2}\sin\theta\right)$ — ②

$\hat{k}$: $\vec{M}_A = \sum \vec{r}_{i/A} \times \vec{F}_i$

$= \left(\frac{L}{2}\sin\theta\,\hat{i} - \frac{L}{2}\cos\theta\,\hat{j}\right) \times (-mg\,\hat{j})$

$= -\frac{mgL}{2}\sin\theta\,\hat{k}$

$I_G\vec{\alpha}_G + \vec{r}_{G/A} \times m\vec{a}_G$

$= \frac{mL^2}{12}\alpha_{AB}\hat{k} + \left[\left(\frac{L}{2}\sin\theta\,\hat{i} - \frac{L}{2}\cos\theta\,\hat{j}\right) \times m\left[\left(a_x + \frac{\alpha_{AB}L}{2}\cos\theta\right)\hat{i} + \frac{\alpha_{AB}L}{2}\sin\theta\,\hat{j}\right]\right]$

$= \frac{mL^2}{12}\alpha_{AB}\hat{k} + m\left[\frac{L^2}{4}\alpha_{AB}\sin^2\theta\,\hat{k} + \frac{a_x L}{2}\cos\theta\,\hat{k} + \frac{L^2\alpha_{AB}}{4}\cos^2\theta\,\hat{k}\right]$

$= \left[\frac{mL^2\alpha_{AB}}{12} + \frac{mL^2\alpha_{AB}}{4} + \frac{ma_x L}{2}\cos\theta\right]\hat{k}$

$= \left(\frac{mL^2\alpha_{AB}}{3} + \frac{ma_x L}{2}\cos\theta\right)\hat{k}$

$$-\frac{mgL}{2}\sin\theta = \frac{mL^2}{3}\alpha_{AB} + \frac{ma_x L}{2}\cos\theta$$

$$A_y - mg = \frac{m\alpha_{AB}L}{2}\sin\theta$$

$$0 = m\left(a_x + \frac{\alpha_{AB}L}{2}\cos\theta\right)$$

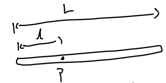
$$a_x = -\frac{\alpha_{AB}L}{2}\cos\theta$$

$$-\frac{mgL}{2}\sin\theta = \frac{mL^2\alpha_{AB}}{3} + \frac{mL}{2}\cos\theta \times \left(-\frac{\alpha_{AB}L}{2}\cos\theta\right)$$

$$-6g\sin\theta = 4L\alpha_{AB} - 3L\alpha_{AB}\cos^2\theta$$

$$\alpha_{AB} = \frac{-6g\sin\theta}{L(4-3\cos^2\theta)} \quad //$$

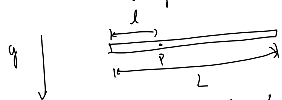
$$a_x = \frac{6g\sin\theta\cos\theta}{2(4-3\cos^2\theta)} \quad //$$



m, L

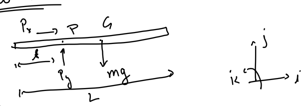
release from rest.

compute 'l' so α_{bar} is maximum.



l so that α_{Bar} is max after releasing from rest.

1) <u>Draw FBD</u>



2) $\bar{a}_G$, α_{bar}, ω_{bar}

$$\vec{a}_G = \cancel{\vec{a}_P}^{\,0} + \vec{a}_{G/P}$$

$$= \vec{\alpha}_{bar} \times \vec{r}_{G/P} - \cancel{\omega^2_{bar}}^{\,0}\, \vec{r}_{G/P}$$

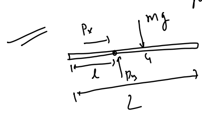
$$= \alpha_{bar}\,\hat{k} \times \left(\frac{L}{2} - l\right)\hat{i}$$

$$= \alpha_{bar}\left(\frac{L}{2} - l\right)\hat{j}$$

3) Choose a point 'p' about which to compute torques.

$$\vec{M}_P = I_G\vec{\alpha}_{bar} + \vec{r}_{G/P} \times m\vec{a}_G$$

if $P \equiv G$, $\vec{M}_G = I_G\vec{\alpha}_{bar}$



4) Write Newton-Euler equations

$\Sigma F_{x\,\&\,y} = m a_{x\,\&\,y}$ ✓

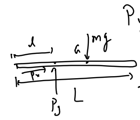
$\vec{M}_G = I_G\alpha_{bar}$ ✓

$\vec{a}_G = \alpha_{bar}\left(\frac{L}{2} - l\right)\hat{j}$ $\qquad I = \frac{ml^2}{12}$

$$P_x = m a_x = 0 \quad -①$$

$$P_y - mg = m a_y$$

$$P_y - mg = m\alpha_{bar}\left(\frac{L}{2} - l\right) \quad -②$$



$\vec{M}_G = ?$

$M_{mg} = M_{P_x} = 0$

$\vec{P}_y = P_y\,\hat{j}$

$\vec{r}_{P/G} = -\left(\frac{L}{2} - l\right)\hat{i}$

$$\vec{M}_{P_y} = \vec{r}_{P/G} \times \vec{P}_y = -\left(\frac{L}{2} - l\right)P_y\,\hat{k}$$

$$-\left(\frac{L}{2} - l\right)P_y = \frac{mL^2}{12}\alpha_{bar} \quad -③$$

$$P_y - mg = m\alpha_{bar}\left(\frac{L}{2} - l\right)$$

$$\Rightarrow P_y = mg + m\alpha_{bar}\left(\frac{L}{2} - l\right) \quad -④$$

$3 \times 2 \times 2$

$$-\left(\frac{L}{2} - l\right)\left(\cancel{m}g + \cancel{m}\,\alpha_{bar}\left(\frac{L}{2} - l\right)\right) = \frac{\cancel{m}L^2}{12}\alpha_{bar}$$

$\times 2$

$$\Rightarrow -3(L - 2l)\left(2g + \alpha_{bar}(L - 2l)\right) = L^2\alpha_{bar}$$

$$\Rightarrow -6(L - 2l)g = \left(L^2 + 3(L - 2l)^2\right)\alpha_{bar}$$

$$\Rightarrow -6(L - 2l)g = \left(L^2 + 3L^2 + 12l^2 - 12lL\right)\alpha_{bar}$$

$$\Rightarrow -6(L - 2l)g = \left(4L^2 + 12l^2 - 12lL\right)\alpha_{bar}$$

$$\Rightarrow \alpha_{bar} = \frac{3(2l - L)g}{2(L^2 - 3Ll + 3l^2)}$$

$$\alpha_{bar} = \frac{3g(2l - L)}{2(L^2 - 3lL + 3l^2)}$$

$$\frac{d\alpha_{bar}}{dl} = 0$$

$$y = \frac{f(x)}{g(x)}, \quad \frac{dy}{dx}$$

$$y = f(x)g(x)$$

$$\frac{dy}{dx} = f'(x)g(x) + g'(x)f(x)$$

$$y = \frac{f(x)}{g(x)}$$

$$\frac{dy}{dx} = \frac{f'(x)}{g(x)} - \frac{1}{g(x)^2}g'(x)f(x)$$

$$\frac{dy}{dx} = \frac{g(x)f'(x) - g'(x)f(x)}{g(x)^2} \quad //$$

$$\alpha_{bar} = \frac{3(2l - L)}{2(L^2 - 3Ll + 3l^2)}$$

$$\frac{d\alpha_{bar}}{dl} = \frac{2(L^2 - 3Ll + 3l^2)(6) - (-6L + 12l)3(2l-L)}{(\quad)^2} = 0$$

$$\Rightarrow 2(L^2 - 3Ll + 3l^2)\cancel{6} + \cancel{6}(L - 2l)3(2l - L) = 0$$

$$\Rightarrow \begin{array}{l} 2L^2 - 6Ll + 6l^2 \\ -3L^2 + 12lL - 12l^2 = 0 \end{array}$$

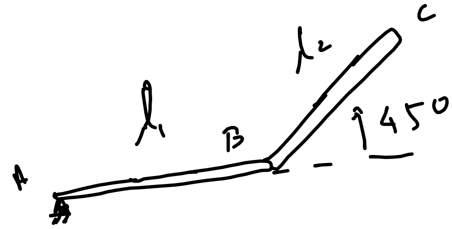
$$\Rightarrow -L^2 + 6Ll - 6l^2 = 0$$

$$\Rightarrow 6l^2 - 6Ll + L^2 = 0$$

$$l = \frac{6 \pm \sqrt{36 - 4 \times 6}}{2 \times 6} L$$

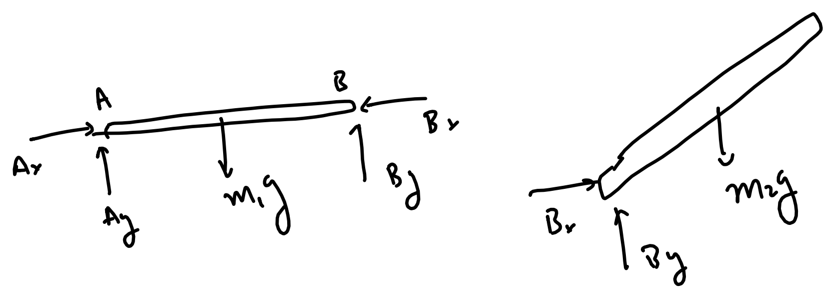
$$= \frac{3 \pm \sqrt{3}}{6} L$$

$$\alpha_{bar} = \pm\frac{\sqrt{3}}{L}g \quad //$$



m_1, m_2

$\alpha_{BC}, \alpha_{AB}, A_x, A_y$ 4 unknowns



$A_x, A_y, B_x, B_y, \alpha_{AB}, \alpha_{BC}$

$$\sum F_x = \ldots$$
$$\sum F_y = \ldots$$
$$\sum M = \ldots$$

both bodies