

$$\tau = I\alpha$$

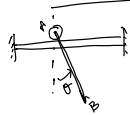


$$\vec{v}_P = \vec{v}_A + \vec{\omega} \times \vec{r}_{P/A}$$

$$\vec{a}_P = \vec{a}_A + \vec{\alpha} \times \vec{r}_{P/A} - \omega^2 \vec{r}_{P/A}$$



- 1) Draw FBD
- 2) Choose a point 'p' to compute torque/moment about
- 3) Compute  $\vec{a}_c$
- 4)  $\vec{F} = m\vec{a}_c$  (2. eqns for planar case)
- 5)  $\vec{M}_p = \sum \vec{r}_{i/p} \times \vec{F}_i = I\vec{\alpha}_c + \vec{r}_{c/p} \times m\vec{a}_c$  (1. eqn for planar)



$$I_c = \frac{mL^2}{12}$$

$\alpha_A = ?$   $\alpha_{AB} = ?$  Right after release



$$\begin{aligned} \vec{a}_c &= \vec{a}_A + \vec{a}_{c/A} \\ &= \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{c/A} - \omega_{AB}^2 \vec{r}_{c/A} \\ &= a_A \hat{j} \\ &\quad + \alpha_{AB} \hat{k} \times \left( \frac{L}{2} \sin\theta \hat{i} - \frac{L}{2} \cos\theta \hat{j} \right) \\ &\quad - \omega_{AB}^2 \left( \frac{L}{2} \sin\theta \hat{i} - \frac{L}{2} \cos\theta \hat{j} \right) \end{aligned}$$

$$\begin{aligned} \vec{a}_c &= a_A \hat{j} + \alpha_{AB} \frac{L}{2} \sin\theta \hat{j} + \frac{\alpha_{AB} L}{2} \cos\theta \hat{i} \\ &\quad - \omega_{AB}^2 \frac{L}{2} \sin\theta \hat{i} + \frac{\omega_{AB}^2 L}{2} \cos\theta \hat{j} \\ &= \left( a_A + \frac{\alpha_{AB} L}{2} \cos\theta - \omega_{AB}^2 \frac{L}{2} \sin\theta \right) \hat{i} \\ &\quad + \left( \frac{\alpha_{AB} L}{2} \sin\theta + \omega_{AB}^2 \frac{L}{2} \cos\theta \right) \hat{j} \end{aligned}$$

As AB is released from rest,  $\omega_{AB} = 0$ .

$$\vec{a}_c = \left( a_A + \frac{\alpha_{AB} L}{2} \cos\theta \right) \hat{i} + \frac{\alpha_{AB} L}{2} \sin\theta \hat{j}$$



$$\begin{aligned} \sum \vec{F} &= m\vec{a}_c \text{ in } x \text{ \& } y \text{ directions} \\ \vec{M}_A &= I_A \vec{\alpha}_{AB} + \vec{r}_{c/A} \times m\vec{a}_c \\ \vec{M}_c &= I_c \vec{\alpha}_{AB} \end{aligned}$$

$$\sum F_x = ma_x \Rightarrow 0 = m \left( a_A + \frac{\alpha_{AB} L}{2} \cos\theta \right) \quad \text{--- (1)}$$

$$\sum F_y = ma_y \Rightarrow A_j - mg = m \left( \frac{\alpha_{AB} L}{2} \sin\theta \right) \quad \text{--- (2)}$$

$$\begin{aligned} \vec{M}_A &= \sum \vec{r}_{i/A} \times \vec{F}_i \\ &= \left( \frac{L}{2} \sin\theta \hat{i} - \frac{L}{2} \cos\theta \hat{j} \right) \times (-mg \hat{j}) \\ &= -\frac{mgL}{2} \sin\theta \hat{k} \end{aligned}$$

$$\begin{aligned} I_A \vec{\alpha}_c + \vec{r}_{c/A} \times m\vec{a}_c &= \frac{mL^2}{12} \alpha_{AB} \hat{k} + \left( \frac{L}{2} \sin\theta \hat{i} - \frac{L}{2} \cos\theta \hat{j} \right) \\ &\quad \times m \left[ \left( a_A + \frac{\alpha_{AB} L}{2} \cos\theta \right) \hat{i} + \frac{\alpha_{AB} L}{2} \sin\theta \hat{j} \right] \end{aligned}$$

$$= \frac{mL^2}{12} \alpha_{AB} \hat{k} + \left[ \frac{L^2}{4} \alpha_{AB} \sin^2\theta \hat{k} + \frac{\alpha_{AB} L}{2} \cos\theta \hat{k} - \frac{L^2}{4} \alpha_{AB} \sin\theta \cos\theta \hat{k} \right]$$

$$= \left[ \frac{mL^2}{12} \alpha_{AB} + \frac{mL^2}{4} \alpha_{AB} + \frac{m\alpha_{AB} L}{2} \cos\theta \right] \hat{k}$$

$$= \left( \frac{mL^2}{3} \alpha_{AB} + \frac{m\alpha_{AB} L}{2} \cos\theta \right) \hat{k}$$

$$-\frac{mgL}{2} \sin \theta = \frac{mL^2}{3} \alpha_{AB} + \frac{ma_x L}{2} \cos \theta$$

$$A_y - mg = \frac{m \alpha_{AB} L}{2} \sin \theta$$

$$0 = m \left( a_x + \frac{\alpha_{AB} L}{2} \cos \theta \right)$$

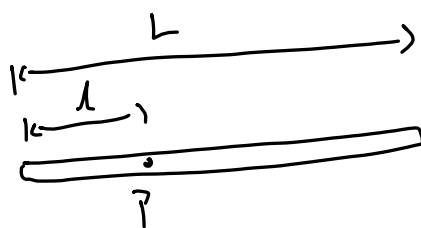
$$a_x = - \frac{\alpha_{AB} L}{2} \cos \theta$$

$$-\frac{mgL}{2} \sin \theta = \frac{mL^2}{3} \alpha_{AB} + \frac{mL}{2} \cos \theta \left( -\frac{\alpha_{AB} L}{2} \cos \theta \right)$$

$$-6g \sin \theta = 4L \alpha_{AB} - 3L \alpha_{AB} \cos^2 \theta$$

$$\alpha_{AB} = \frac{-6g \sin \theta}{L(4 - 3 \cos^2 \theta)} //$$

$$a_x = \frac{6g \sin \theta \cos \theta}{2(4 - 3 \cos^2 \theta)} //$$



$m, L$

release from rest.

compute 'l' so  $\alpha_{bar}$  is maximum.