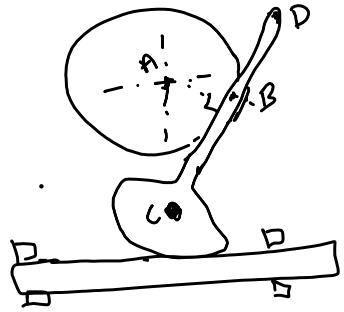
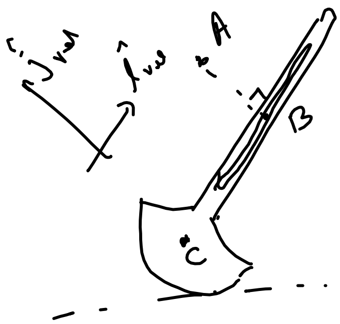
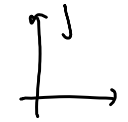


find ω_{CD} when $AB \perp CD$.

$\vec{r}_{B/C} = r_{B/C}\,\hat{i}_{rel}$





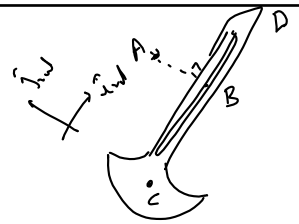
$$\vec{V}_B^{A} = \cancel{\vec{V}_A}^{0} + \vec{V}_{B/A}$$
$$= \vec{\omega}_{AB} \times \vec{r}_{B/A}$$
$$= \omega_{AB}\hat{k} \times r_{B/A}(-\hat{j}_{rel})$$
$$= \omega_{AB} r_{B/A}\,\hat{i}_{rel}$$

$$\vec{V}_B^{C} = \vec{V}_C + \vec{V}_{B/C}$$
$$= \vec{0} + \dot{\vec{\rho}}_{rel} + \vec{\omega} \times \vec{\rho}_{rel}$$
$$= \dot{r}_{B/C}\,\hat{i}_{rel} + (\omega_{CD}\hat{k} \times r_{B/C}\hat{i}_{rel})$$
$$= \dot{r}_{B/C}\,\hat{i}_{rel} + \omega_{CD} r_{B/C}\,\hat{j}_{rel}$$

equating velocity from A & C origins

$$\vec{V}_B^{A} = \vec{V}_B^{C}$$
$$\Rightarrow \omega_{AB} r_{B/A}\,\hat{i}_{rel} = \dot{r}_{B/C}\,\hat{i}_{rel} + \omega_{CD} r_{B/C}\,\hat{j}_{rel}$$
$$\Rightarrow \dot{r}_{B/C} = \omega_{AB} r_{B/A}$$
& $0 = \omega_{CD} r_{B/C}$

$$\Rightarrow \omega_{CD} = 0$$



$\to$ calculate $\vec{a}_B^c$ & $\vec{a}_B^A$

$\to$ equate $\vec{a}_B^c$ to $\vec{a}_B^A$

$\to$ solve

$$\vec{a}_B^A = \vec{a}_A + \vec{a}_{B/A}$$
$$= \vec{0} + \vec{\alpha}_{AB} \times \vec{r}_{D/A} + \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{B/A})$$
$$= \cancel{\vec{\alpha}_{AB}}^{0} \times \vec{r}_{B/A} - |\vec{\omega}_{AB}|^2 \vec{r}_{B/A}$$
$$= -\omega_{AB}^2 r_{B/A} (-\hat{j}_{rel})$$
$$= \omega_{AB}^2 r_{B/A} \hat{j}_{rel}$$

$$\vec{a}_B^c = \vec{a}_c + \vec{a}_{B/c} + 2(\vec{\omega} \times \dot{\vec{\rho}}_{rel}) + \vec{\alpha} \times \vec{\rho}_{rel}$$
$$= \vec{0} + \ddot{\vec{\rho}}_{rel} + 2(\vec{\omega} \times \dot{\vec{\rho}}_{rel}) + \vec{\omega} \times (\vec{\omega} \times \vec{\rho}_{rel})$$

$$= \underbrace{\ddot{\vec{\rho}}_{rel}}_{\text{Radial}} + \underbrace{2(\vec{\omega} \times \dot{\vec{\rho}}_{rel})}_{\text{Coriolis}} + \underbrace{\vec{\alpha} \times \vec{\rho}_{rel}}_{\text{Tangential}} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{\rho}_{rel})}_{\text{Centripetal}}$$

$$= \ddot{r}_{B/c} \hat{i}_{rel} + 2(\omega_{CD} \hat{k} \times \dot{r}_{B/c} \hat{i}_{rel}) + \omega_{CD} \hat{k} \times (\omega_{CD} \hat{k} \times r_{B/c} \hat{i}_{rel}) + \alpha_{CD} \hat{k} \times r_{B/c} \hat{i}_{rel}$$

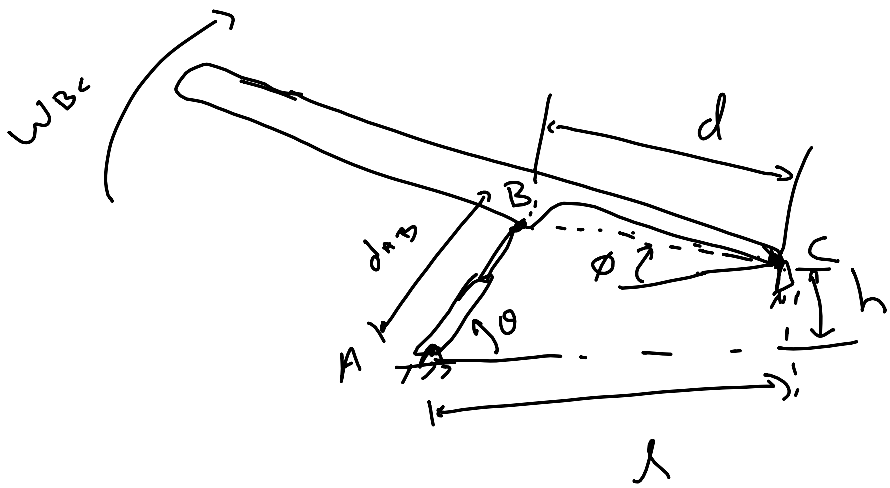
$$= \ddot{r}_{B/c} \hat{i}_{rel} + 2\omega_{CD} \dot{r}_{B/c} \hat{j}_{rel} + \alpha_{CD} r_{B/c} \hat{j}_{rel} - \omega_{CD}^2 r_{B/c} \hat{i}_{rel}$$

$$= (\ddot{r}_{B/c} - \omega_{CD}^2 r_{B/c}) \hat{i}_{rel} + (2\omega_{CD} \dot{r}_{B/c} + \alpha_{CD} r_{B/c}) \hat{j}_{rel}$$

$\omega_{CD} = 0$
$\Rightarrow$
$$\vec{a}_B^c = \ddot{r}_{B/c} \hat{i}_{rel} + \alpha_{CD} r_{B/c} \hat{j}_{rel}$$
$$\vec{a}_B^A = \omega_{AB}^2 r_{B/A} \hat{j}_{rel}$$

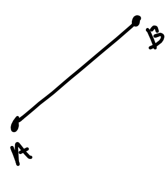
equating $\hat{i}$ & $\hat{j}$ components

$$\ddot{r}_{B/c} = 0$$
$$\alpha_{CD} r_{B/c} = \omega_{AB}^2 r_{B/A}$$
$$\Rightarrow \boxed{\alpha_{CD} = \frac{\omega_{AB}^2 r_{B/A}}{r_{B/c}}}$$



find $\dot{d}_{AB}$ when $\phi = 0$, $\ddot{d}_{AB}$

1) No arrow for vectors



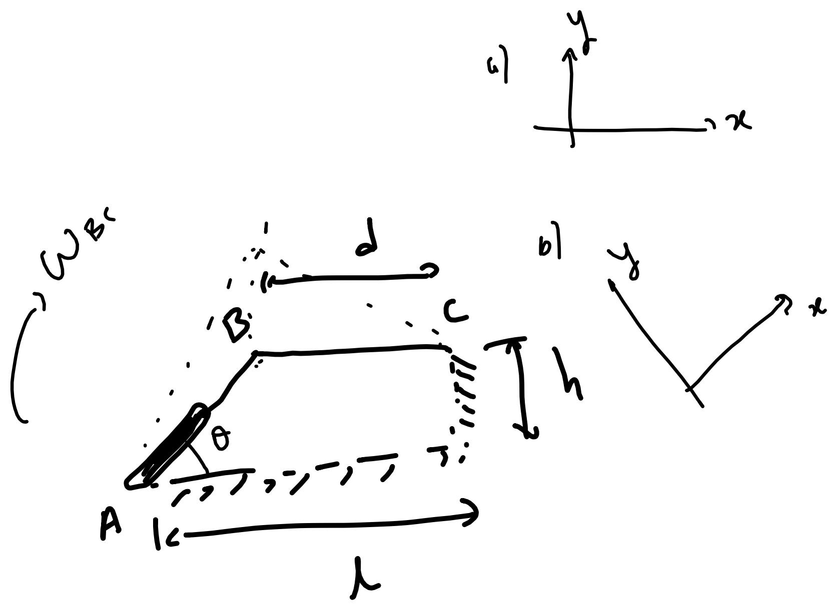
$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \quad \Big| \quad r_A = r_B + r_{B/A}$$

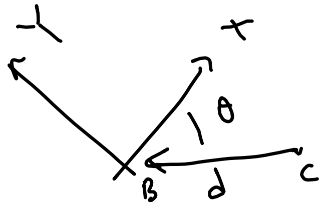
$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$

$$= \vec{V}_A + \dot{r}_{B/A}\,\hat{u}_{r_{B/A}} + \vec{\omega} \times \vec{r}_{B/A}$$

2) Not drawing axis

3) Not drawing simplified scenario.





$$\vec{V}_B^c = \vec{V}_c + \vec{V}_{B/c}$$

$$= \vec{0} + \cancel{\dot{r}_{B/c}}^{\,0}\,\hat{u}_{r_{B/c}} + \vec{\omega}_{BC} \times \vec{r}_{B/c}$$

$$= \vec{\omega}_{BC} \times \vec{r}_{B/c}$$

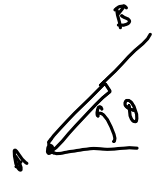
$$= (-\omega_{BC}\hat{k}) \times (-d\cos\theta\,\hat{i} + d\sin\theta\,\hat{j})$$

$$= (-\omega_{BC}\hat{k}) \times (-d\cos\theta\,\hat{i}) + (-\omega_{BC}\hat{k}) \times (d\sin\theta\,\hat{j})$$

$$= \omega_{BC} d\cos\theta(\hat{k} \times \hat{i}) - \omega_{BC} d\sin\theta(\hat{k} \times j)$$

$$= \omega_{BC} d\cos\theta\,\hat{j} + \omega_{BC} d\sin\theta\,\hat{i}$$

$$\vec{V}_B^c = \omega_{BC} d\sin\theta\,\hat{i} + \omega_{BC} d\cos\theta\,\hat{j}$$



$$\vec{V}_B^{A} = \vec{V}_A + \vec{V}_{B/A}$$
$$= \vec{0} + \dot{r}_{B/A}\hat{u}_{r_{B/A}} + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$
$$= \vec{0} + \dot{\vec{\rho}}_{B/A,rel} + \vec{\omega}_{AB} \times \vec{\rho}_{B/A,rel}$$

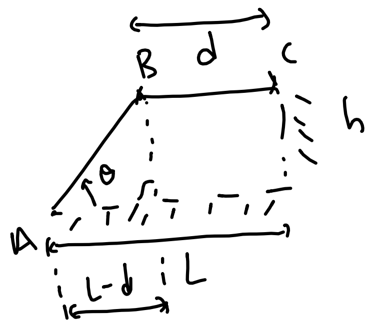
$$\vec{V}_B^{A} = \dot{r}_{B/A}\hat{u}_{B/A} + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$
$$= \dot{d}_{AB}\,\hat{i} + (\omega_{AB}\hat{k}) \times (d_{AB}\hat{i})$$

$$\vec{V}_B^{A} = \dot{d}_{AB}\,\hat{i} + \omega_{AB} d_{AB}\,\hat{j}$$

$$\vec{V}_D^{C} = \omega_{BC} d \sin\theta\,\hat{i} + \omega_{BC} d\cos\theta\,\hat{j}$$

$$\dot{d}_{AB} = \omega_{BC} d \sin\theta$$

$$\omega_{AB} = \frac{\omega_{BC} d\cos\theta}{d_{AB}}$$



$$\tan\theta = \frac{h}{L-d}, \quad \cos\theta = \frac{L-d}{\sqrt{h^2 + (L-d)^2}}$$

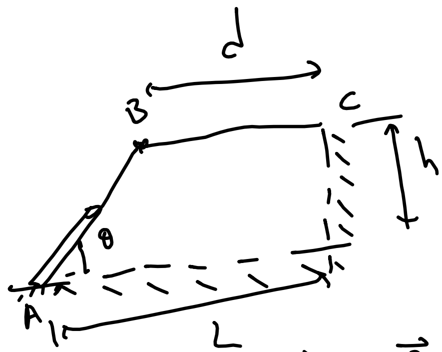
$$\sin\theta = \frac{h}{\sqrt{h^2 + (L-d)^2}}, \quad d_{AB} = \sqrt{h^2 + (L-d)^2}$$

$$\dot{d}_{AB} = \omega_{BC} d \sin\theta$$

$$\dot{d}_{AB} = \frac{\omega_{BC} d\, h}{\sqrt{h^2 + (L-d)^2}}$$

$$\omega_{AB} = \frac{\omega_{BC} d\cos\theta}{d_{AB}} = \frac{\omega_{BC} d (L-d)}{h^2 + (L-d)^2} \quad //$$

$$\ddot{d}_{AB}\ , \quad \omega_{AB} = \frac{\omega_{BC}\, d(L-d)}{h^2+(L-d)^2}$$



$$\vec{a}_B^{\,C}$$

$$\vec{a}_B^{\,A}$$

$$\vec{a}_B^{\,C} = \vec{a}_C + \vec{a}_{B/C}$$
$$= \vec{0} + \ddot{r}_{B/C}\,\hat{u}_{r\,B/C} + \vec{\alpha}_{BC}\times\vec{r}_{B/C}$$
$$+ \vec{\omega}_{BC}\times(\vec{\omega}_{BC}\times\vec{r}_{B/C})$$
$$+ 2\dot{r}_{B/C}(\vec{\omega}_{BC}\times\vec{u}_{r\,B/C})$$

As $r_{B/C}$ is constant,

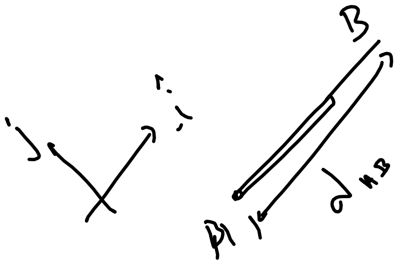
$$\vec{a}_B^{\,C} = \vec{\alpha}_{BC}\times\vec{r}_{B/C} + \vec{\omega}_{BC}\times(\vec{\omega}_{BC}\times\vec{r}_{B/C})$$

As $\alpha_{BC}=0$,

$$\vec{a}_B^{\,C} = \vec{\omega}_{BC}\times(\vec{\omega}_{BC}\times\vec{r}_{B/C})$$

As $\vec{\omega}_{BC}\perp\vec{r}_{B/C}$,

$$\vec{a}_B^{\,C} = -|\vec{\omega}_{BC}|^2\,\vec{r}_{B/C}$$
$$= -\omega_{BC}^2\,\vec{r}_{B/C}$$



$$\vec{a}_B^A = \ddot{r}_{B/A}\hat{u}_{r_{B/A}} + \vec{\alpha}_{AB}\times\vec{r}_{B/A} + \vec{\omega}_{AB}\times(\vec{\omega}_{AB}\times\vec{r}_{B/A}) + 2\dot{r}_{B/A}(\vec{\omega}_{AB}\times\hat{u}_{r_{B/A}})$$

$$\vec{a}_B^A = \ddot{d}_{AB}\hat{i} + (\alpha_{AB}\hat{k}\times d_{AB}\hat{i}) - \omega_{AB}^2(d_{AB}\hat{i}) + 2\dot{d}_{AB}(\omega_{AB}\hat{k}\times\hat{i})$$

$$= (\ddot{d}_{AB} - \omega_{AB}^2 d_{AB})\hat{i} + (\alpha_{AB}d_{AB} + 2\dot{d}_{AB}\omega_{AB})\hat{j}$$

$$\vec{a}_B^C = -\omega_{BC}^2\vec{r}_{B/C} = -\omega_{BC}^2(-d\cos\theta\,\hat{i} + d\sin\theta\,\hat{j})$$

Equating 'i' components

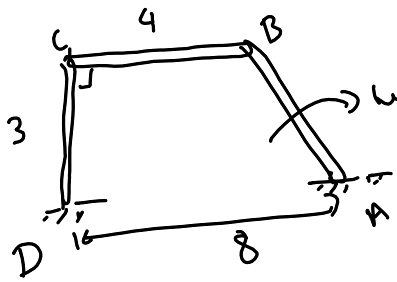
$$\ddot{d}_{AB} - \omega_{AB}^2 d_{AB} = \omega_{BC}^2 d\cos\theta$$

$$\ddot{d}_{AB} = \omega_{BC}^2 d\cos\theta + \omega_{AB}^2 d_{AB}$$

$$= \frac{\omega_{BC}^2 d(L-d)}{\sqrt{h^2+(L-d)^2}} + \left(\frac{\omega_{BC} d(L-d)}{h^2+(L-d)^2}\right)^2\sqrt{h^2+(L-d)^2}$$

$$= \frac{\omega_{BC}^2 d(L-d)}{\sqrt{h^2+(L-d)^2}}\left[1+\frac{d(L-d)}{h^2+(L-d)^2}\right]$$

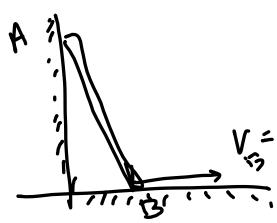
I.C.

1) 

$\omega_{AB} = 1 \text{rad s}^{-1}$

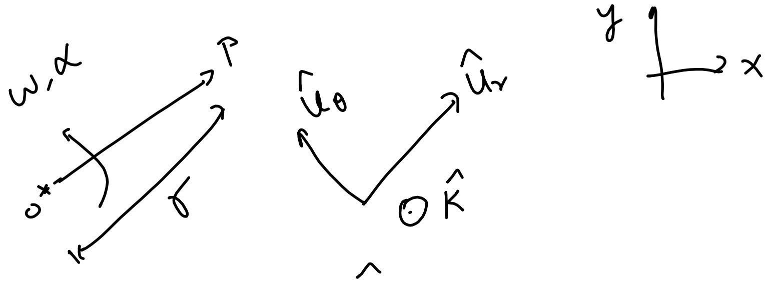
$\alpha_{AB} = 0$

compute $\vec{V}_c$, $\vec{a}_c$, α_{BC}, ω_{BC}

2) 

calculate, $\vec{V}_A$, ω_{AB}, α_{AB}

$\vec{a}_A$

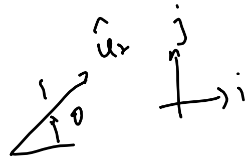


$$\vec{r} = r\,\hat{u}_r$$

$$\vec{V} = \dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{d(r\,\hat{u}_r)}{dt}$$

$$\frac{d(fg)}{dx} = f\frac{dg}{dx} + g\frac{df}{dx} \qquad = \frac{dr}{dt}\hat{u}_r + r\frac{d\hat{u}_r}{dt}$$

$$= \dot{r}\,\hat{u}_r$$



$$\hat{u}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\frac{d\hat{u}_r}{dt} = -\sin\theta\,\dot{\theta}\,\hat{i} + \cos\theta\,\dot{\theta}\,\hat{j}$$

$$y = h(z) \qquad \frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx}$$
$$z = f(x) \qquad = \dot{h}(z)\,\dot{f}(x)$$

$$\frac{d\hat{u}_r}{dt} = -\sin\theta\,\dot{\theta}\,\hat{i} + \cos\theta\,\dot{\theta}\,\hat{j}$$
$$= \sin\theta\,\dot{\theta}(\hat{k}\times\hat{j}) + \cos\theta\,\dot{\theta}\,(\hat{k}\times\hat{i})$$
$$= \dot{\theta}\,\hat{k}\times(\sin\theta\,\hat{j} + \cos\theta\,\hat{i})$$
$$= \vec{\omega}\times\hat{u}_r$$

$$\vec{V} = \dot{r}\,\hat{u}_r + r\frac{d\hat{u}_r}{dt} = \dot{r}\,\hat{u}_r + r(\vec{\omega}\times\hat{u}_r)$$
$$\vec{V} = \dot{r}\,\hat{u}_r + \vec{\omega}\times\vec{r}$$

April 20, 2017

$\dot{\hat{u}}_r = \vec{\omega} \times \hat{u}_r$

$$\vec{V} = \dot{r}\,\hat{u}_r + \vec{\omega} \times \vec{r}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d(\dot{r}\hat{u}_r + \vec{\omega}\times\vec{r})}{dt}$$

$$= \frac{d(\dot{r}\hat{u}_r)}{dt} + \frac{d(\vec{\omega}\times\vec{r})}{dt}$$

$$= \frac{d\dot{r}}{dt}\hat{u}_r + \dot{r}\frac{d\hat{u}_r}{dt} + \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\vec{r}}{dt}$$

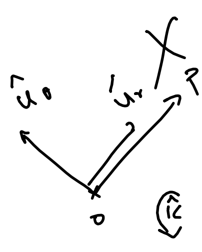
$$= \ddot{r}\,\hat{u}_r + \dot{r}(\vec{\omega}\times\hat{u}_r) + \vec{\alpha}\times\vec{r} + \vec{\omega}\times(\dot{r}\,\hat{u}_r + \vec{\omega}\times\vec{r})$$

$$= \underbrace{\ddot{r}\,\hat{u}_r}_{\text{radial } \cancel{\text{vel}} \text{ acc}} + \underbrace{2\dot{r}(\vec{\omega}\times\hat{u}_r)}_{\text{coriolis}} + \underbrace{\vec{\alpha}\times\vec{r}}_{\text{tangential}} + \underbrace{\vec{\omega}\times(\vec{\omega}\times\vec{r})}_{\text{centripetal}}$$

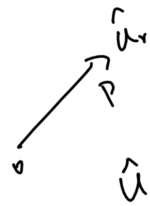
1) $\vec{\omega} \perp \vec{r}$, $\omega \to \hat{k}$, $\vec{r} \to \hat{u}_r, \hat{u}_\theta, \hat{i}, \hat{j}$

$$\vec{a} = \ddot{r}\,\hat{u}_r + 2\dot{r}(\omega\hat{k}\times\hat{u}_r) + \alpha\hat{k}\times r\hat{u}_r + \omega\hat{k}\times(\omega\hat{k}\times r\hat{u}_r)$$

$$\boxed{\begin{aligned} \vec{a} &= (\ddot{r} - r\omega^2)\hat{u}_r + (\alpha r + 2\dot{r}\omega)\hat{u}_\theta \\ \vec{V} &= \dot{r}\,\hat{u}_r + r\omega\,\hat{u}_\theta \end{aligned}}$$

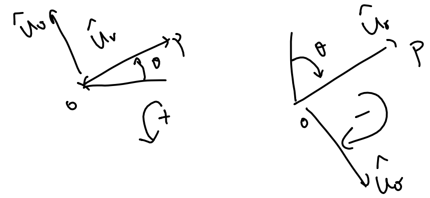


$\curvearrowleft +\omega$ $\curvearrowright +ve$



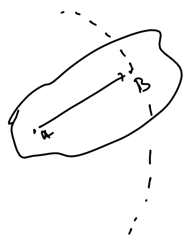
$\hat{k} \to$ arc tonation

$\hat{u}_\theta \to \hat{k}\times\hat{u}_r$ ✓



I.C. → Property of rigid body

→ distance between '2' points is fixed



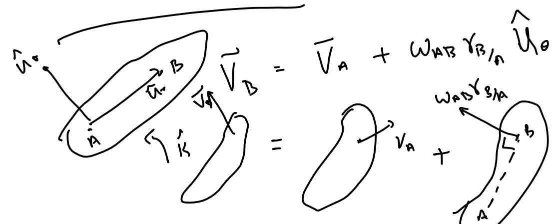
$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$

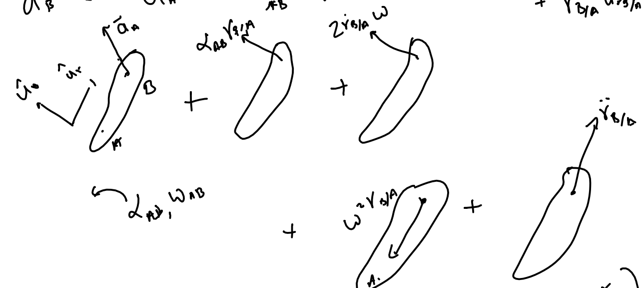
$$= \vec{V}_A + \cancel{\dot{r}_{B/A}}^{\,0}\,\hat{u}_{r_{B/A}} + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

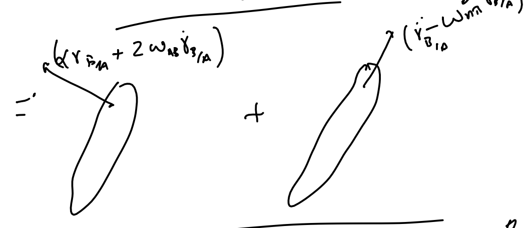
$$= \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$\vec{V}_B = \vec{V}_A + \omega_{AB}\, r_{B/A}\, \hat{u}_{\theta_{B/A}}$$



$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} + 2\dot{r}_{B/A}(\vec{\omega}_{AB} \times \vec{r}_{B/A}) + \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{B/A}) + \ddot{r}_{B/A}\,\hat{u}_{r_{B/A}}$$





---

for planar rigid bodies

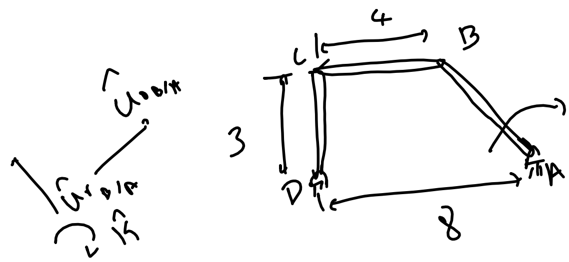
$$\vec{a}_B = \vec{a}_A + (\cancel{\ddot{r}_{B/A}}^{\,0} - r_{B/A}\omega_{AB}^2)\,\hat{u}_{r_{B/A}} + (\alpha r_{B/A} + \cancel{2\omega \dot{r}_{B/A}}^{\,0})\,\hat{u}_{\theta_{B/A}}$$

$$= \vec{a}_A - r_{B/A}\,\omega_{AB}^2\,\hat{u}_{r_{B/A}} + \alpha r_{B/A}\,\hat{u}_{\theta_{B/A}}$$

I.C.

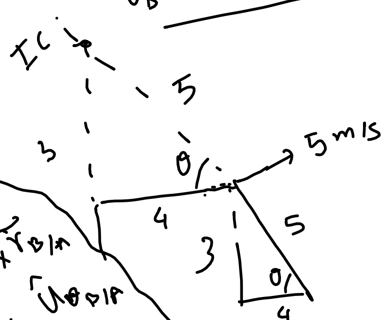
$$\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

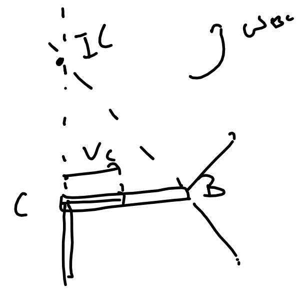
Point on rigid body wher $\vec{V} = 0$ is called instantaneous center.



$\omega = 1$ rad/s
$\alpha = 0$, ω_{BC} & $\vec{V}_C$

$$V_B^A = \omega_{AB} r_{B/A} = 1 \times 5 = 5 \text{ m/s}$$





Nothing to do with question

$\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$= \vec{V}_A + \omega_{AB} r_{B/A} \hat{u}_{\theta B/A}$

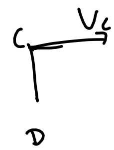
If A is IC, $\vec{V}_{IC} = 0$

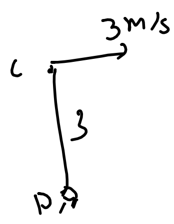
$\vec{V}_B = \omega_{AB} r_{B/IC} \hat{u}_{\theta B/IC}$

$$\omega_{BC} = \frac{V_B}{R_{IC,B}} = \frac{V_C}{R_{IC,C}}$$

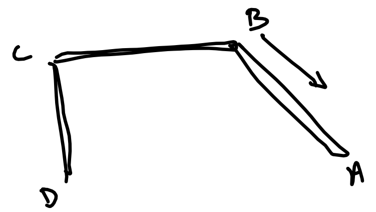
$$\omega_{BC} = \frac{5}{5} = \frac{V_C}{3}$$

$\omega_{BC} = 1$, $V_C = 3$ m/s





$$\Rightarrow \omega_{CD} = \frac{V_C}{R_{CD}} = \frac{3}{3} = 1 \text{ rad/s}$$

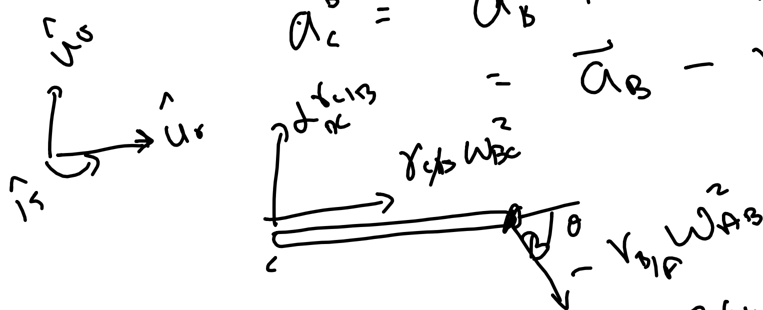


$\omega_{AB} = \omega_{BC} = \omega_{CD} = 1\ \text{rad/s}$

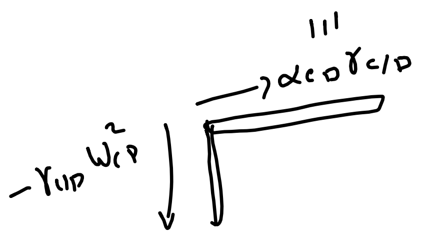
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$
$$= \vec{a}_A - r_{B/A}\omega_{AB}^2 \hat{u}_{r_{B/A}} + \alpha_{AB} r_{B/A} \hat{u}_{\theta_{B/A}}$$

$$a_B^n = \searrow -r_{AB}\omega_{AB}^2$$

$$a_C^B = \vec{a}_B + \vec{a}_{C/B}^{B}$$
$$= \vec{a}_B - r_{C/B}\omega_{BC}^2 \hat{u}_{r_{C/B}} + \alpha_{BC} r_{C/B} \hat{u}_{\theta_{C/B}}$$



equate acc in 'y' dir



$$\alpha_{BC} r_{C/B} - r_{C/D}\omega_{CD}^2$$
$$= -r_{B/A}\omega_{AB}^2 \sin\theta$$

$$\alpha_{BC}(4) - 3\cdot 1^2 = -\cancel{5} \times 1^2 \times \frac{3}{\cancel{5}}$$

$$\alpha_{BC} = 0$$