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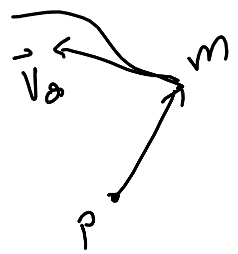
5.3 Linear momentum, $\vec{p} = m\vec{v}_o$



Angular momentum

$$\vec{h}_p = \vec{r}_{o/p} \times m\vec{v}_o$$

$$\frac{d\vec{h}_p}{dt} = \frac{d(\vec{r}_{o/p} \times m\vec{v}_o)}{dt}$$

$$= \frac{d\vec{r}_{o/p}}{dt} \times m\vec{v}_o + \vec{r}_{o/p} \times m\frac{d\vec{v}_o}{dt}$$

$$= \vec{v}_{o/p} \times m\vec{v}_o + \vec{r}_{o/p} \times \vec{F}$$

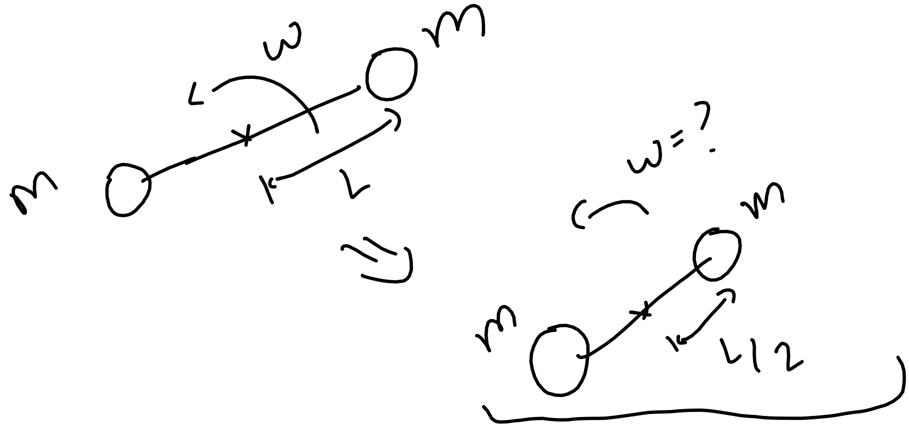
$$= (\vec{v}_o - \vec{v}_p) \times m\vec{v}_o + \vec{r}_{o/p} \times \vec{F}$$

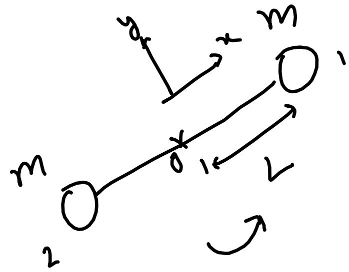
$$= -\vec{v}_p \times m\vec{v}_o + \vec{r}_{o/p} \times \vec{F}$$

$$\vec{r}_{o/p} \times \vec{F} = \dot{\vec{h}}_p + \vec{v}_p \times m\vec{v}_o$$

if $\vec{v}_p = 0$ or $\vec{v}_p \times m\vec{v}_o = 0$

$$\vec{M}_o = \vec{r}_{o/p} \times \vec{F} = \dot{\vec{h}}_p$$





$$\vec{V}_1 = L\omega\hat{j}$$

$$\vec{V}_2 = -L\omega\hat{j}$$

$$\vec{r}_{1/0} = L\hat{i}, \quad \vec{r}_{2/0} = -L\hat{i}$$

$$\begin{aligned}\vec{h}_0 &= \sum \vec{r}_{i/0} \times \vec{p}_i \\ &= \vec{r}_{1/0} \times m\vec{V}_1 + \vec{r}_{2/0} \times m\vec{V}_2 \\ &= m(L\hat{i}) \times (L\omega\hat{j}) + m(-L\hat{i}) \times (-L\omega\hat{j}) \\ &= 2mL^2\omega\hat{k}\end{aligned}$$

1) compute $\vec{h}_P$ for 2nd case

2) what is ω_2?

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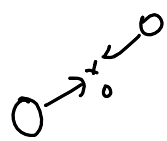
$$\bar{h}_o = 2mL^2\omega\hat{k}$$

case 1 , case 2

$L_1 = L$, $L_2 = L/2$

$\omega_1 = \omega_1$, $\omega_2 = ?$



As there is no friction

$\vec{M}_f = 0$

As int forces pass through 'O',

$\vec{M}_{int} = 0$

$\vec{M} = 0 \Rightarrow \vec{h}_P$ is constant

$$2mL_1^2\omega_1 = 2mL_2^2\omega_2$$

$$2mL^2\omega = 2m\left(\frac{L}{2}\right)^2\omega_2$$

$$\Rightarrow \boxed{\omega_2 = 4\omega}$$

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