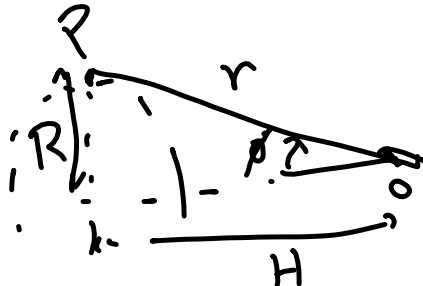


$$\vec{V}_P = \dot{R} \hat{u}_R + R \dot{\theta} \hat{u}_\theta$$

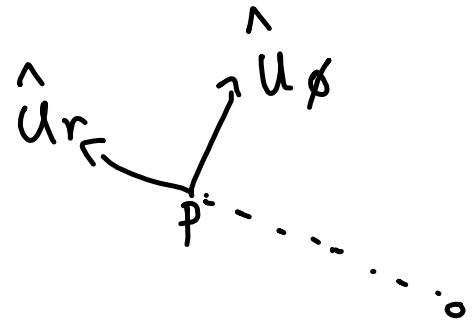
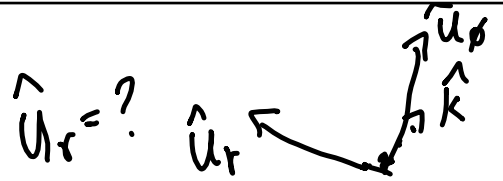
$$\vec{V}_P = R \dot{\theta} \hat{u}_\theta$$

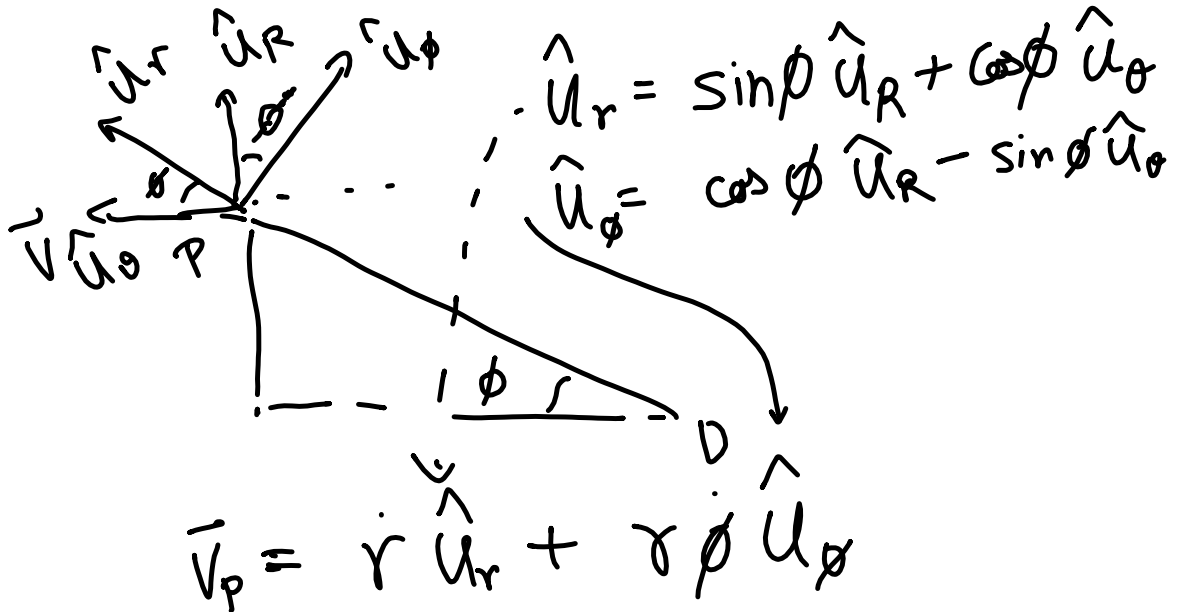


$$\vec{v}_p = \dot{r} \hat{u}_r + r \dot{\phi} \hat{u}_\phi$$

?                      ?

$$r = \sqrt{R^2 + H^2} \quad \checkmark$$

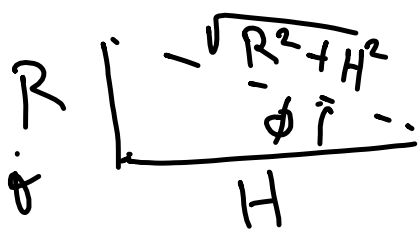




$$\begin{aligned}
 \vec{v}_p &= \dot{r} (\sin \phi \hat{u}_R + \cos \phi \hat{u}_\theta) \\
 &\quad + r \dot{\phi} (\cos \phi \hat{u}_R - \sin \phi \hat{u}_\theta) \\
 &= (\dot{r} \sin \phi + r \dot{\phi} \cos \phi) \hat{u}_R \\
 &\quad + (\dot{r} \cos \phi - r \dot{\phi} \sin \phi) \hat{u}_\theta \\
 &= 0 \hat{u}_R + R \dot{\theta} \hat{u}_\theta
 \end{aligned}$$

$$\dot{r} \sin \phi + r \dot{\phi} \cos \phi = 0$$

$$\dot{r} \cos \phi - r \dot{\phi} \sin \phi = R \dot{\theta}$$



$$\cos \phi = \frac{H}{\sqrt{R^2 + H^2}}$$

$$\sin \phi = \frac{R}{\sqrt{R^2 + H^2}}$$

$$\dot{r} \sin \phi + r \dot{\phi} \cos \phi = 0$$

$$\Rightarrow \dot{r} \frac{R}{\sqrt{R^2 + H^2}} + r \dot{\phi} \frac{H}{\sqrt{R^2 + H^2}} = 0$$

$$\Rightarrow \dot{r} R + r \dot{\phi} H = 0$$

$$\Rightarrow \dot{r} = -\frac{r}{R} H \dot{\phi}$$

$$\dot{r} \cos \phi - r \dot{\phi} \sin \phi = R \dot{\theta}$$

$$R \left( -\frac{r}{R} \dot{\phi} \frac{H}{\sqrt{R^2+H^2}} - \frac{r \dot{\phi} R}{\sqrt{R^2+H^2}} = R \dot{\theta} \right)$$

$$-(H^2 + R^2) \dot{\phi} = R^2 \dot{\theta}$$

$$\Rightarrow \dot{\phi} = -\frac{R^2 \dot{\theta}}{R^2 + H^2}$$



$$\dot{\phi} = - \frac{R^2 \dot{\theta}}{R^2 + H^2}$$

$$\dot{\gamma} = - \frac{rH}{R} \dot{\phi} = - \frac{\cancel{\sqrt{R^2 + H^2}} H}{R} \times \frac{-R^2 \dot{\theta}}{\sqrt{R^2 + H^2}}$$

$$\dot{\gamma} = \frac{RH}{\sqrt{R^2 + H^2}} \dot{\theta}$$

$$\dot{\phi} = \frac{-R^2 \dot{\theta}}{R^2 + H^2} = \frac{-(1.5)^2 \times 2\pi}{(1.5^2 + 4^2)}$$

$$\dot{\phi} = -0.7746 \text{ rad/s}$$

$$\dot{r} = \frac{RH \dot{\theta}}{\sqrt{R^2 + H^2}} \quad \dot{\theta} = \frac{4 \times 1.5 \times 2\pi}{\sqrt{1.5^2 + 4^2}}$$

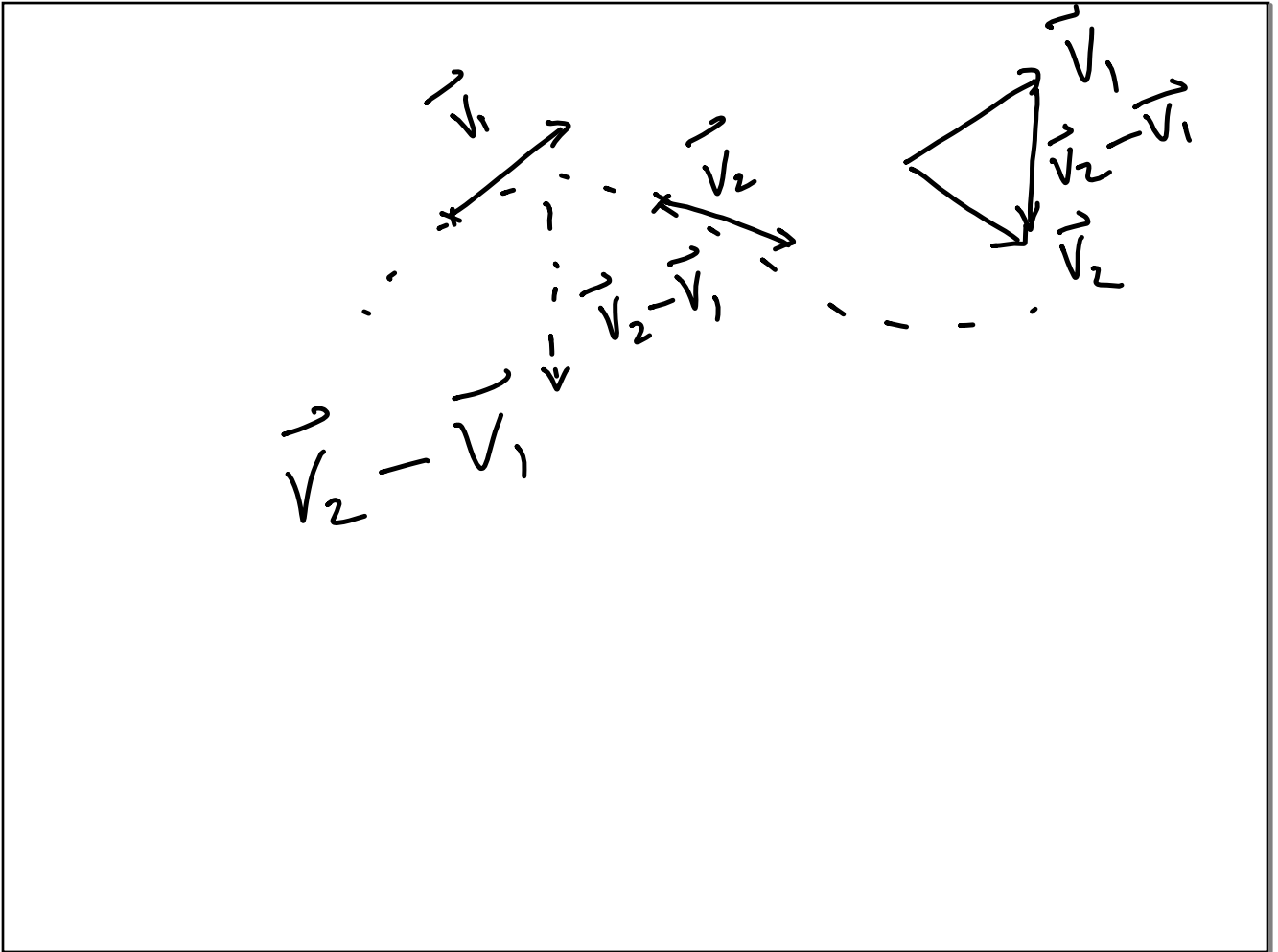
$$\dot{r} = 18.825 \text{ ft/s}$$

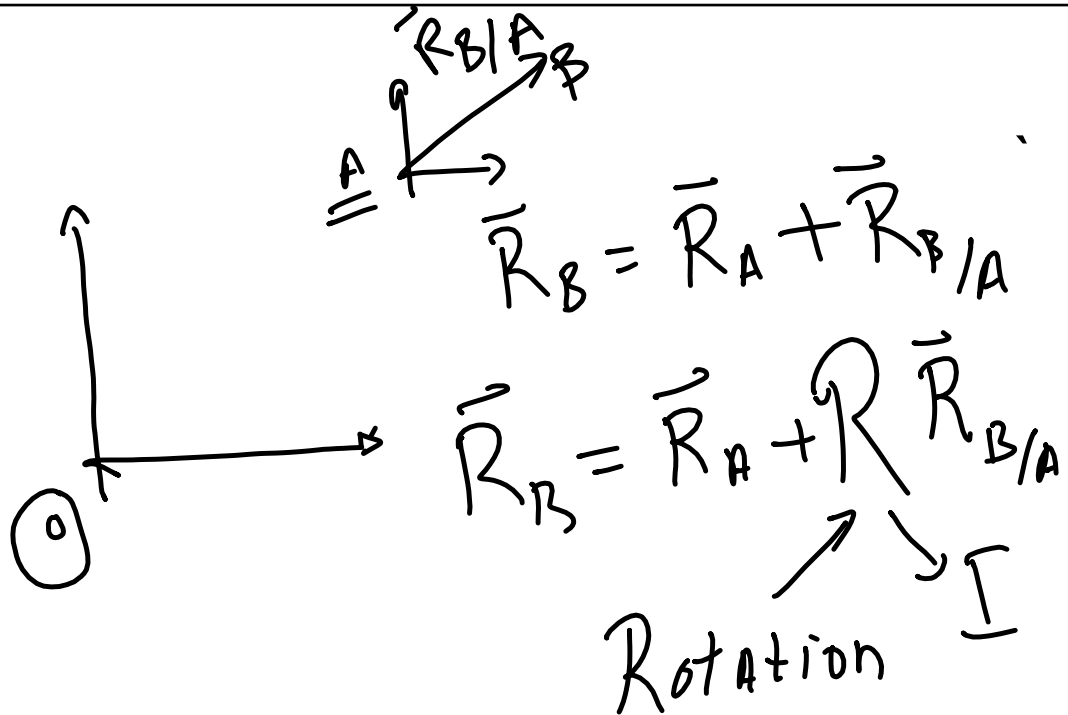
$$60 \text{ rpm} = 60 \frac{\text{rev}}{\text{min}} = \frac{60 \times 2\pi}{60} \frac{\text{rad}}{\text{s}} = 2\pi \text{ rad/s}$$

The diagram shows a circle with a point on its circumference. A vertical arrow points downwards from the center of the circle to the point. A horizontal arrow points to the left from the point. A longer arrow points from the center of the circle to the point. To the right of the circle, the variables  $\dot{r}$  and  $\ddot{\phi}$  are written with horizontal arrows pointing to the right. Below the circle, the acceleration vector  $\vec{a}_p$  is expressed in polar coordinates as:

$$\vec{a}_p = (\ddot{r} - r\dot{\phi}^2)\hat{u}_r + (2\dot{r}\dot{\phi} + \ddot{r}r)\hat{u}_\phi$$

There are question marks above  $\ddot{r}$  and below  $\ddot{r}r$  in the original image. A checkmark is placed under  $\dot{\phi}$  in the second term. A downward arrow is next to  $\hat{u}_\phi$ . To the left of the circle, the expression  $\vec{a}_p = -R\dot{\theta}^2\hat{u}_r$  is written.





February 7, 2017

