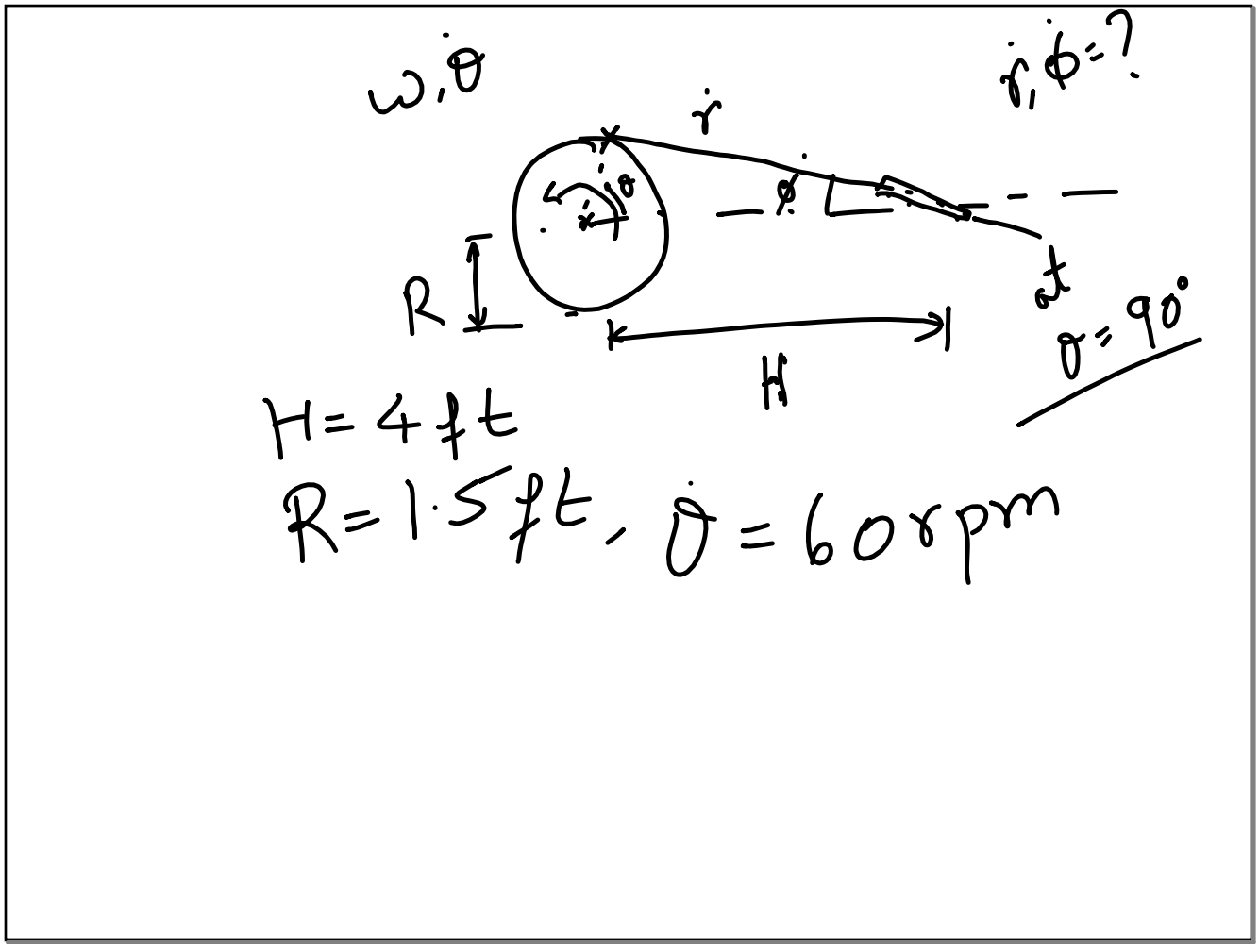
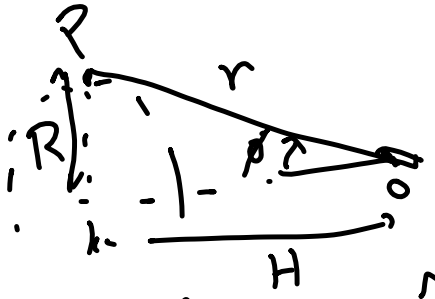
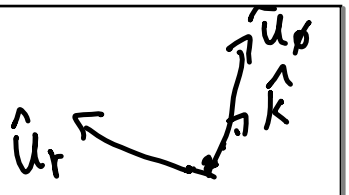

ω, θ̇
ṙ, φ̇ = ?
r
θ
φ
R
H
at
θ = 90°
H = 4 ft
R = 1.5 ft, θ̇ = 60 rpm

$$\vec{V}_P = \cancel{\dot{R}}\,\hat{u}_R + R\dot{\theta}\,\hat{u}_\theta$$

$$\vec{V}_P = R\dot{\theta}\,\hat{u}_\theta$$

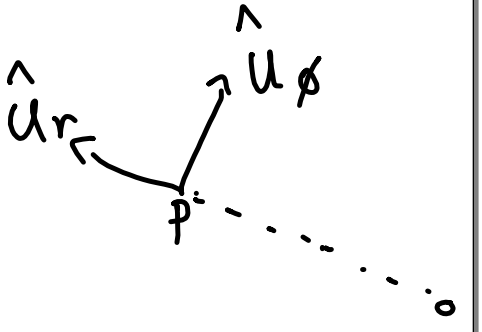


$\hat{u}_r = ?$

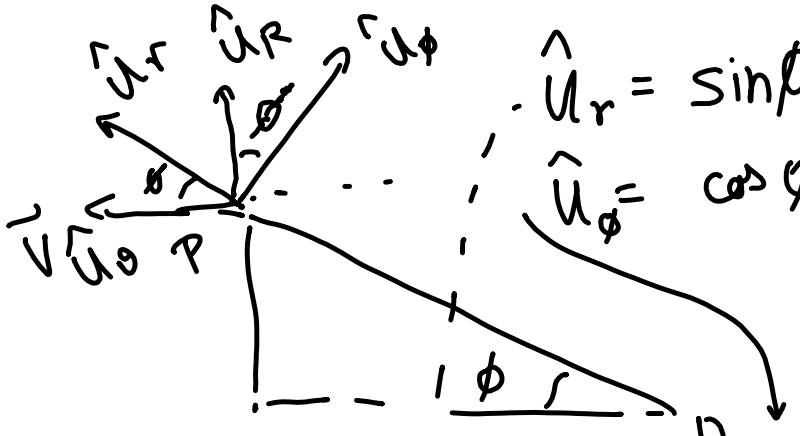


$$\vec{v}_p = \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta$$

? ?



$$r = \sqrt{R^2 + H^2}$$ ✓



$$\hat{u}_r = \sin\phi\,\hat{u}_R + \cos\phi\,\hat{u}_\theta$$

$$\hat{u}_\phi = \cos\phi\,\hat{u}_R - \sin\phi\,\hat{u}_\theta$$

$$\vec{V}_p = \dot{r}\,\hat{u}_r + r\dot{\phi}\,\hat{u}_\phi$$

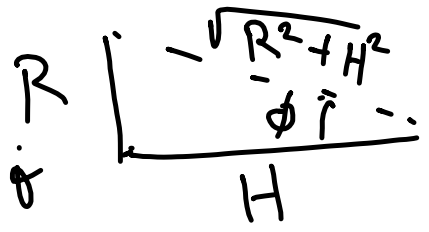
$$\vec{v}_P = \dot{r}(\sin\phi\,\hat{u}_R + \cos\phi\,\hat{u}_\theta) + r\dot{\phi}(\cos\phi\,\hat{u}_R - \sin\phi\,\hat{u}_\theta)$$

$$= (\dot{r}\sin\phi + r\dot{\phi}\cos\phi)\hat{u}_R + (\dot{r}\cos\phi - r\dot{\phi}\sin\phi)\hat{u}_\theta$$

$$= 0\,\hat{u}_R + R\dot{\theta}\,\hat{u}_\theta$$

$$\dot{r}\sin\phi + r\dot{\phi}\cos\phi = 0$$

$$\dot{r}\cos\phi - r\dot{\phi}\sin\phi = R\dot{\theta}$$



$$\cos\phi = \frac{H}{\sqrt{R^2+H^2}}$$

$$\sin\phi = \frac{R}{\sqrt{R^2+H^2}}$$

$$\dot{r}\sin\phi + r\dot{\phi}\cos\phi = 0$$

$$\Rightarrow \dot{r}\frac{R}{\sqrt{R^2+H^2}} + r\dot{\phi}\frac{H}{\sqrt{R^2+H^2}} = 0$$

$$\Rightarrow \dot{r}R + r\dot{\phi}H = 0$$

$$\Rightarrow \dot{r} = -\frac{r}{R}H\dot{\phi}$$

$$\dot{r}\cos\phi - r\dot{\phi}\sin\phi = R\dot{\theta}$$

$$R\left(-\frac{H}{R}\dot{\phi}\frac{H}{\sqrt{R^2+H^2}} - r\dot{\phi}\frac{R}{\sqrt{R^2+H^2}} = R\dot{\theta}\right)$$

$$-(H^2+R^2)\dot{\phi} = R^2\dot{\theta}$$

$$\Rightarrow \quad \dot{\phi} = -\frac{R^2\dot{\theta}}{R^2+H^2}$$

$$\dot{\phi} = -\frac{R^2\dot{\theta}}{R^2+H^2}$$

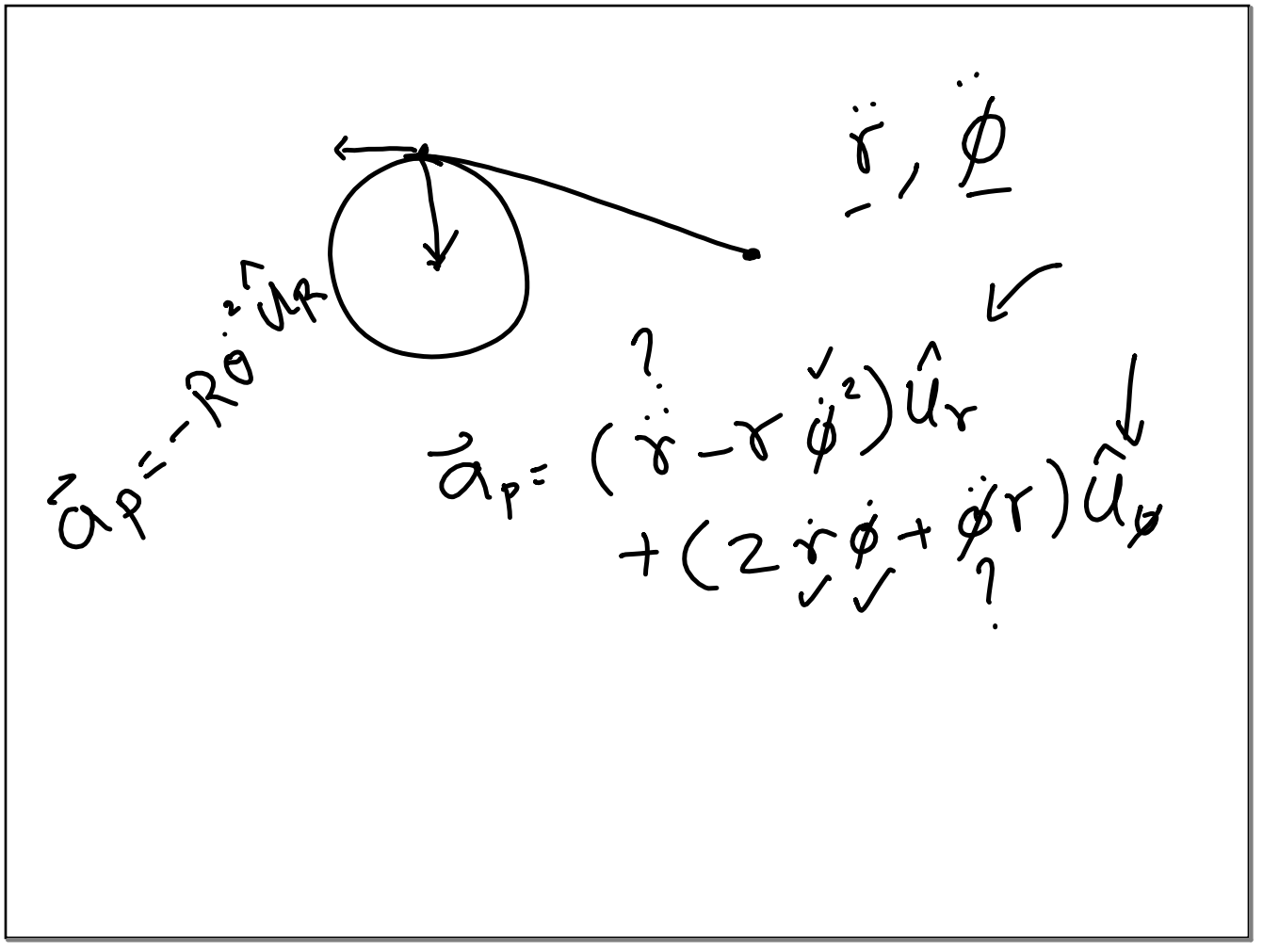
$$\dot{r} = -\frac{rH}{R}\dot{\phi} = -\frac{\sqrt{R^2+H^2}}{R}H \times \frac{-R^2\dot{\theta}}{\sqrt{R^2+H^2}}$$

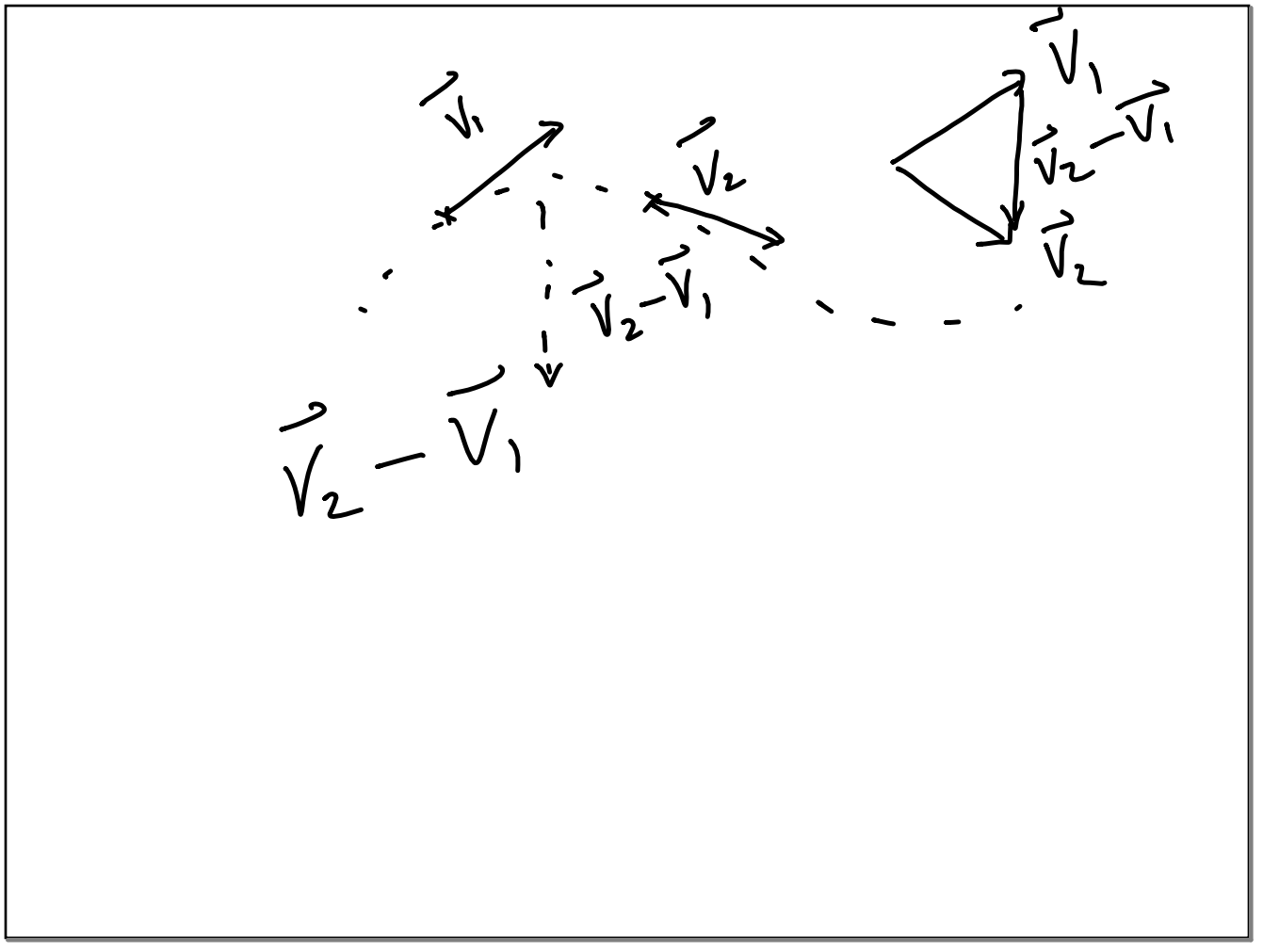
$$\dot{r} = \frac{RH}{\sqrt{R^2+H^2}}\dot{\theta}$$

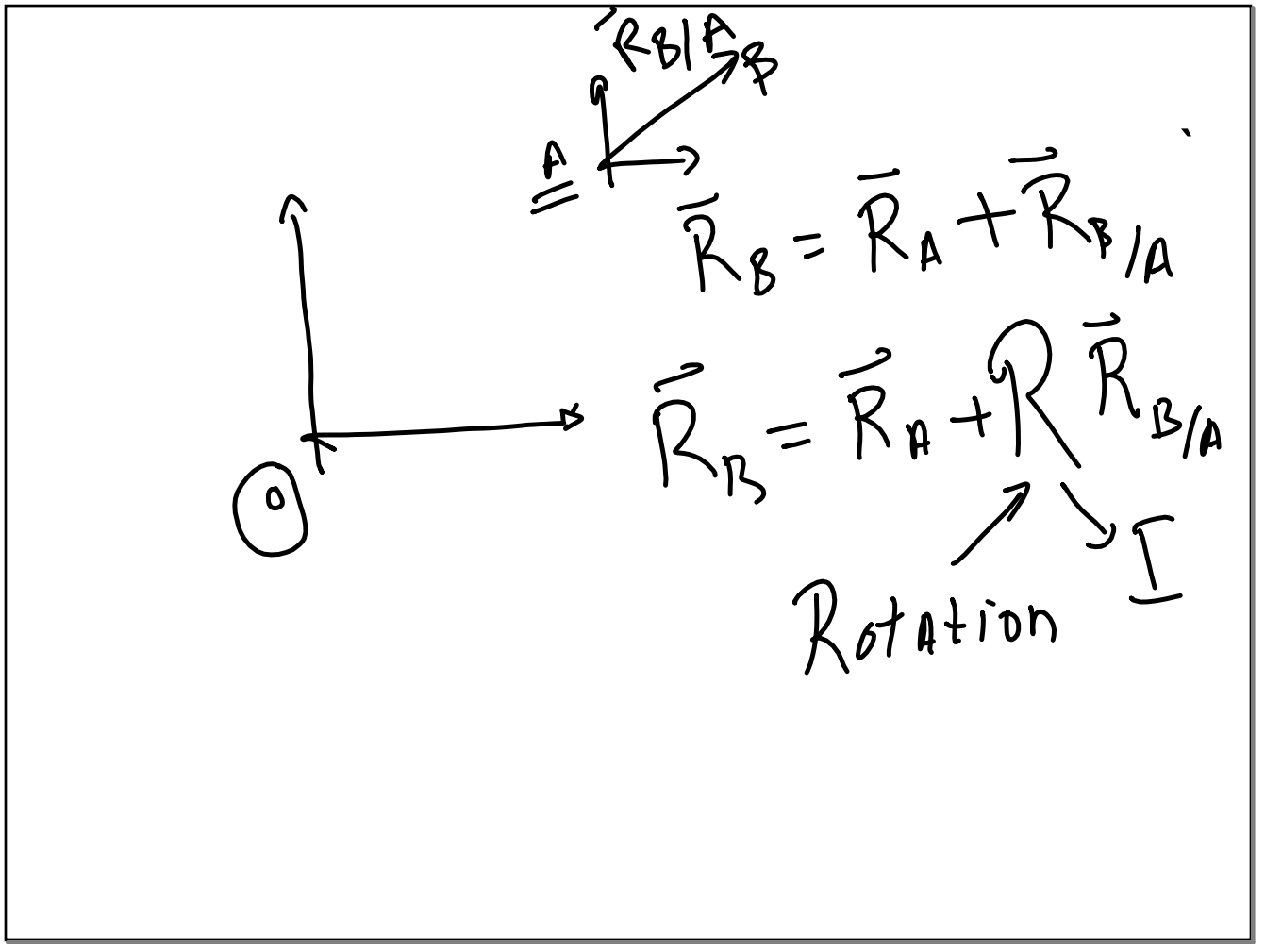
$$\dot{\phi} = \frac{-R^2\dot{\theta}}{R^2+H^2} = \frac{-(1.5)^2 \times 2\pi}{(1.5^2+4^2)} = -.7746 \text{ rad/s}$$

$$\dot{r} = \frac{RH}{\sqrt{R^2+H^2}}\dot{\theta} = \frac{4 \times 1.5 \times 2\pi}{\sqrt{1.5^2+4^2}} = 3.825 \text{ ft/s}$$

$$60 \text{ rpm} = 60 \frac{\text{rev}}{\text{min}} = \frac{60 \times 2\pi}{60} \frac{\text{rad}}{\text{s}} = 2\pi \text{ rad/s}$$


$\ddot{r}, \ddot{\phi}$
$\vec{a}_p = -R\dot{\theta}^2 \hat{u}_R$
$\vec{a}_p = (\ddot{r} - r\dot{\phi}^2)\hat{u}_r + (2\dot{r}\dot{\phi} + \ddot{\phi}r)\hat{u}_\theta$


$\vec{V}_1$
$\vec{V}_2$
$\vec{V}_2 - \vec{V}_1$
$\vec{V}_1$
$\vec{V}_2$
$\vec{V}_2 - \vec{V}_1$
$\vec{V}_2$
$\vec{V}_2 - \vec{V}_1$


$\vec{R}_{B/A}$
B
A
O
$\vec{R}_B = \vec{R}_A + \vec{R}_{B/A}$
$\vec{R}_B = \vec{R}_A + R\,\vec{R}_{B/A}$
Rotation
I

February 7, 2017