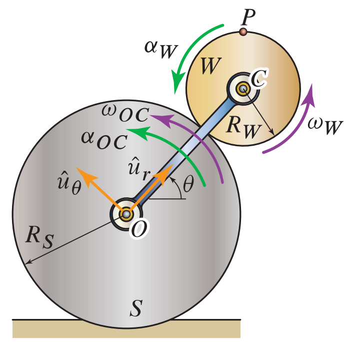
Problem 6.121

A wheel W of radius $R_W = 2$ in. rolls without slip over the stationary cylinder S of radius $R_S = 5$ in., and the wheel is connected to point O via the arm OC.

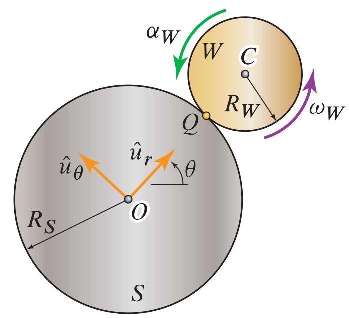
Determine the acceleration of the point on the wheel W that is in contact with S for $\omega_{OC} = 7.5$ rad/s and $\alpha_{OC} = 2$ rad/s^2.



Solution

Relating the velocity of C to that of O, we obtain

$$\vec{v}_C = \vec{v}_O + \vec{\omega}_{OC} \times \vec{r}_{C/O} = \omega_{OC}\,\hat{k} \times (R_S + R_W)\,\hat{u}_r = (R_S + R_W)\omega_{OC}\,\hat{u}_\theta,$$



where we have used the fact that O is a fixed point. Now let Q be the point on W in contact with S. Since W rolls without slip on S, $\vec{v}_Q = \vec{0}$, and so

$$\vec{v}_C = \vec{v}_Q + \vec{\omega}_W \times \vec{r}_{C/Q} = \omega_W\,\hat{k} \times R_W\,\hat{u}_r = R_W\omega_W\,\hat{u}_\theta.$$

Equating these two expressions for $\vec{v}_C$, we obtain the angular velocity of the wheel as

$$(R_S + R_W)\omega_{OC}\,\hat{u}_\theta = R_W\omega_W\,\hat{u}_\theta \quad\Rightarrow\quad \omega_W = \frac{R_S + R_W}{R_W}\omega_{OC}. \tag{1}$$

Relating the acceleration of C to that of O, we have

$$\vec{a}_C = \vec{a}_O + \vec{\alpha}_{OC} \times \vec{r}_{C/O} - \omega_{OC}^2\vec{r}_{C/O} = \alpha_{OC}\,\hat{k} \times (R_S + R_W)\,\hat{u}_r - \omega_{OC}^2(R_S + R_W)\,\hat{u}_r = -(R_S + R_W)\omega_{OC}^2\,\hat{u}_r + (R_S + R_W)\alpha_{OC}\,\hat{u}_\theta, \tag{2}$$

where we have used the fact that point O is fixed. Now relating the acceleration of Q to that of C, we obtain

$$\begin{aligned}\vec{a}_Q &= \vec{a}_C + \vec{\alpha}_W \times \vec{r}_{Q/C} - \omega_W^2\vec{r}_{Q/C}\\ &= -(R_S + R_W)\omega_{OC}^2\,\hat{u}_r + (R_S + R_W)\alpha_{OC}\,\hat{u}_\theta + \alpha_W\,\hat{k} \times (-R_W\,\hat{u}_r) - \omega_W^2(-R_W\,\hat{u}_r)\\ &= \left[-(R_S + R_W)\omega_{OC}^2 + R_W\omega_W^2\right]\hat{u}_r + \left[(R_S + R_W)\alpha_{OC} - R_W\alpha_W\right]\hat{u}_\theta.\end{aligned}$$

where we have used Eq. (2) for $\vec{a}_C$. Since W is rolling without slip over S, the component of acceleration tangent to the two bodies at Q must be equal. Therefore, since S is stationary, it must be true that $a_{Q\theta} = 0$, and therefore, the acceleration of A is given by

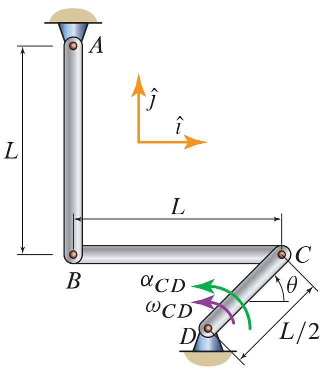
$$\vec{a}_Q = \left[-(R_S + R_W)\omega_{OC}^2 + \frac{(R_S + R_W)^2}{R_W}\omega_{OC}^2\right]\hat{u}_r = \frac{R_S}{R_W}(R_S + R_W)\omega_{OC}^2\,\hat{u}_r,$$

where Eq. (1) was used for ω_W. Using $R_W = 2\text{ in.} = \frac{2}{12}$ ft, $R_S = 5\text{ in.} = \frac{5}{12}$ ft, and $\omega_{OC} = 7.5$ rad/s, we have

$$\boxed{\vec{a}_Q = 82.03\,\hat{u}_r\ \text{ft/s}^2.}$$

Problem 6.125

At the instant shown, bar CD is rotating with an angular velocity 20 rad/s and with angular acceleration 2 rad/s^2 in the directions shown. Furthermore, at this instant $\theta = 45°$. If $L = 2.25$ ft, determine the angular accelerations of bars AB and BC.



Solution

First finding the velocity of C, we obtain

$$\vec{v}_C = \vec{\omega}_{CD} \times \vec{r}_{C/D} = \omega_{CD}\,\hat{k} \times \tfrac{1}{2}L(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}) = \tfrac{1}{2}L\omega_{CD}(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}),$$

where we have enforced the fact that point D is fixed. Now relating the velocity of B to points A and C, we obtain

$$\begin{aligned}
\vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A} &= \vec{v}_C + \vec{\omega}_{BC} \times \vec{r}_{B/C} \\
\Rightarrow \quad \omega_{AB}\,\hat{k} \times (-L\,\hat{\jmath}) &= \tfrac{1}{2}L\omega_{CD}(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}) + \omega_{BC}\,\hat{k} \times (-L\,\hat{\imath}) \\
\Rightarrow \quad L\omega_{AB}\,\hat{\imath} &= -\tfrac{1}{2}L\omega_{CD}\sin\theta\,\hat{\imath} + \left(\tfrac{1}{2}L\omega_{CD}\cos\theta - L\omega_{BC}\right)\hat{\jmath}.
\end{aligned}$$

This equation represents the following two scalar equations in the unknowns ω_{AB} and ω_{BC}:

$$L\omega_{AB} = -\tfrac{1}{2}L\omega_{CD}\sin\theta \quad \text{and} \quad 0 = \tfrac{1}{2}L\omega_{CD}\cos\theta - L\omega_{BC},$$

the solution of which is

$$\omega_{AB} = -\tfrac{1}{2}\omega_{CD}\sin\theta \quad \text{and} \quad \omega_{BC} = \tfrac{1}{2}\omega_{CD}\cos\theta.$$

Proceeding similarly with the acceleration analysis, we begin by finding the acceleration of point C as

$$\begin{aligned}
\vec{a}_C = \vec{\alpha}_{CD} \times \vec{r}_{C/D} - \omega_{CD}^2\vec{r}_{C/D} &= \alpha_{CD}\,\hat{k} \times \tfrac{1}{2}L(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}) - \omega_{CD}^2\left[\tfrac{1}{2}L(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})\right] \\
\Rightarrow \quad \vec{a}_C &= -\tfrac{1}{2}L\left(\alpha_{CD}\sin\theta + \omega_{CD}^2\cos\theta\right)\hat{\imath} + \tfrac{1}{2}L\left(\alpha_{CD}\cos\theta - \omega_{CD}^2\sin\theta\right)\hat{\jmath}.
\end{aligned}$$

We then relate the acceleration of B to points A and C using

$$\vec{a}_B = \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2\vec{r}_{B/A} = \vec{a}_C + \vec{\alpha}_{BC} \times \vec{r}_{B/C} - \omega_{BC}\vec{r}_{B/C},$$

or, substituting in known quantities, we obtain

$$\begin{aligned}
&\alpha_{AB}\,\hat{k} \times (-L\,\hat{\jmath}) - \tfrac{1}{4}\omega_{CD}^2\sin^2\theta(-L\,\hat{\jmath}) \\
&\quad = -\tfrac{1}{2}L\left(\alpha_{CD}\sin\theta + \omega_{CD}^2\cos\theta\right)\hat{\imath} + \tfrac{1}{2}L\left(\alpha_{CD}\cos\theta - \omega_{CD}^2\sin\theta\right)\hat{\jmath} \\
&\qquad + \alpha_{BC}\,\hat{k} \times (-L\,\hat{\imath}) - \tfrac{1}{4}\omega_{CD}^2\cos^2\theta(-L\,\hat{\imath}),
\end{aligned}$$

where we have used the expressions for ω_{AB} and ω_{BC} determined above. Equating components, we have two equations for α_{AB} and α_{BC}:

$$L\alpha_{AB} = -\tfrac{1}{2}L\left(\alpha_{CD}\sin\theta + \omega_{CD}^2\cos\theta\right) + \tfrac{1}{4}L\omega_{CD}^2\cos^2\theta,$$
$$\tfrac{1}{4}L\omega_{CD}^2\sin^2\theta = \tfrac{1}{2}L\left(\alpha_{CD}\cos\theta - \omega_{CD}^2\sin\theta\right) - L\alpha_{BC},$$

the solution of which is

$$\alpha_{AB} = \tfrac{1}{4}\omega_{CD}^2\cos^2\theta - \tfrac{1}{2}\alpha_{CD}\sin\theta - \tfrac{1}{2}\omega_{CD}^2\cos\theta,$$
$$\alpha_{BC} = -\tfrac{1}{4}\omega_{CD}^2\sin^2\theta + \tfrac{1}{2}\alpha_{CD}\cos\theta - \tfrac{1}{2}\omega_{CD}^2\sin\theta.$$

Using $\omega_{CD} = 20\,\text{rad/s}$, $\alpha_{CD} = 2\,\text{rad/s}^2$, $\theta = 45^\circ$, and $L = 2.25\,\text{ft}$, and expressing our answer in vector form, we have

$$\boxed{\vec{\alpha}_{AB} = -92.13\,\hat{k}\,\text{rad/s}^2 \quad \text{and} \quad \vec{\alpha}_{BC} = -190.7\,\hat{k}\,\text{rad/s}^2.}$$