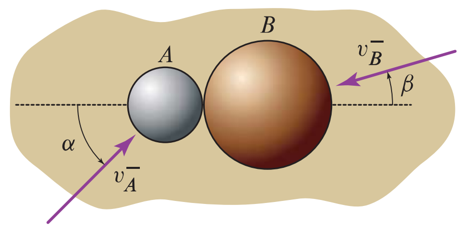
Problem 5.99

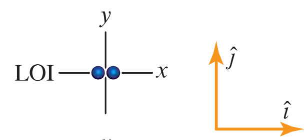
Two spheres, A and B, with masses $m_A = 1.35\,\text{kg}$ and $m_B = 2.72\,\text{kg}$, respectively, collide with $v_A^- = 26.2\,\text{m/s}$, and $v_B^- = 22.5\,\text{m/s}$.

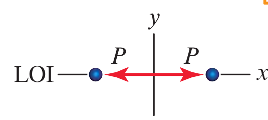
Compute the postimpact velocities of A and B if $\alpha = 45°$, $\beta = 16°$, the COR is $e = 0.57$, and the contact between A and B is frictionless.



Solution

The impact in this problem is a typical two-dimensional oblique central impact. We have selected an xy coordinate system with the x axis aligned with the LOI.





Balance Principles. The impact is therefore characterized by the following three equations:

$$m_A v_{Ax}^- + m_B v_{Bx}^- = m_A v_{Ax}^+ + m_B v_{Bx}^+, \tag{1}$$

$$m_A v_{Ay}^- = m_A v_{Ay}^+, \tag{2}$$

$$m_B v_{By}^- = m_B v_{By}^+. \tag{3}$$

These equations, in order, express the conservation of the linear momentum of the system along the LOI, the conservation of the linear momentum of particle A in the direction normal to the LOI, the conservation of the linear momentum of particle B in the direction normal to the LOI.

Force Laws. The effect of the impulsive contact force P internal to the system is expressed by the COR equation:

$$v_{Ax}^+ - v_{Bx}^+ = e(v_{Bx}^- - v_{Ax}^-). \tag{4}$$

Kinematic Equations. Observe that the preimpact velocities are given and are

$$v_{Ax}^- = v_A^- \cos\alpha, \quad v_{Ay}^- = v_A^- \sin\alpha, \quad v_{Bx}^- = -v_B^- \cos\beta, \quad v_{By}^- = -v_B^- \sin\beta. \tag{5}$$

Computation. Substituting Eqs. (5) into Eqs. (1)–(4) and solving for the components of the postimpact velocities of A and B, we have

$$v_{Ax}^+ = -\frac{1}{m_A + m_B}\left[-m_A v_A^- \cos\alpha + e m_B (v_A^- \cos\alpha + v_B^- \cos\beta) + m_B v_B^- \cos\beta\right], \tag{6}$$

$$v_{Ay}^+ = v_A^- \sin\alpha \tag{7}$$

$$v_{Bx}^+ = -\frac{1}{m_A + m_B}\left[-m_A v_A^- \cos\alpha - e m_A (v_A^- \cos\alpha + v_B^- \cos\beta) + m_B v_B^- \cos\beta\right], \tag{8}$$

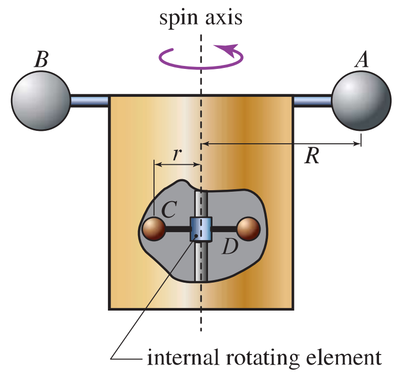
$$v_{By}^+ = -v_B^- \sin\beta, \tag{9}$$

Recalling that $m_A = 1.35\,\text{kg}$, $m_B = 2.72\,\text{kg}$, $v_A^- = 26.2\,\text{m/s}$, $v_B^- = 22.5\,\text{m/s}$, $\alpha = 45°$, $\beta = 16°$, and $e = 0.57$ we can evaluate the velocity components in the the above equations and express the postimpact velocities of A and B to obtain

$$\vec{v}_A^+ = (-23.61\,\hat{\imath} + 18.53\,\hat{\jmath})\,\text{m/s} \quad \text{and} \quad \vec{v}_B^+ = (-0.7174\,\hat{\imath} - 6.202\,\hat{\jmath})\,\text{m/s}.$$

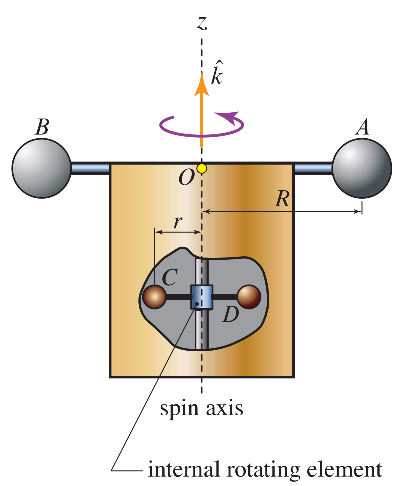
Problem 5.133

The body of the satellite shown has a weight that is negligible with respect to the two spheres A and B that are rigidly attached to it, which weigh 150 lb each. The distance between A and B from the spin axis of the satellite is $R = 3.5$ ft. Inside the satellite there are two spheres C and D weighing 4 lb mounted on a motor that allows them to spin about the axis of the cylinder at a distance $r = 0.75$ ft from the spin axis. Suppose that the satellite is released from rest and that the internal motor is made to spin up the internal masses at an absolute constant time rate of $5.0\,\mathrm{rad/s^2}$ (measured relative to an inertial observer) for a total of 10 s. Treating the system as isolated, determine the angular speed of the satellite at the end of spin-up.



Solution

Referring to the figure at the right, let $\hat{k}$ denote the positive direction of the spin axis and let O be a reference point on the spin axis. We model A, B, C, and D as a system of particles. We observe that this system is isolated. Therefore, the angular momentum of the system is conserved. We define t_1 and t_2 to be the time instants when the C and D are put in motion, and after they have been spun for a total of 10 s, respectively. We use the subscripts 1 and 2 to denote quantities at t_1 and t_2, respectively. We assume that throughout the motion the orientation of the spin axis does not change and that z axis is stationary relative to some inertia reference frame.



Balance Principles. Applying the impulse-momentum principle in the form of conservation of angular momentum, and focusing on the component of the angular momentum along the z axis, which is the spin axis, we have

$$(h_{Oz})_1 = (h_{Oz})_2, \tag{1}$$

where, assuming that the system only rotates about the spins axis,

$$h_{Oz} = (m_A + m_B)\omega_s R^2 + (m_C + m_D)\omega_i r^2, \tag{2}$$

where $\vec{\omega}_s = \omega_s\,\hat{k}$ denotes the angular velocity of the external masses moving with the body of the satellite (the subscript s stands for 'satellite'), and where $\vec{\omega}_i = \omega_i\,\hat{k}$ denotes the angular velocity of the internal masses (the subscript i stands for 'internal').

Force Laws. All forces are accounted for on the FBD (the system is isolated so there are no external forces acting on the system).

Kinematic Equations. Assuming motion of the satellite only about the z axis, the angular acceleration of the internal masses is

$$\vec{\alpha}_i = \alpha_i\,\hat{k}, \tag{3}$$

where $\alpha_i = 5.00\,\mathrm{rad/s^2}$. Assuming that the spin axis does not change orientation, since the system is initially at rest, letting $\tau = t_2 - t_1 = 10$ s, the initial and final angular velocities of the internal masses are

$$\omega_{s1} = 0, \quad \omega_{i1} = 0, \quad \omega_{i2} = \alpha_i\tau. \tag{4}$$

Computation. Substituting Eqs. (2) and Eqs. (4) into Eq. (1), we have

$$0 = (m_A + m_B)R^2\omega_{s2} + (m_C + m_D)r^2\alpha_i\tau \quad \Rightarrow \quad \omega_{s2} = -\frac{(m_C + m_D)r^2}{(m_A + m_B)R^2}\alpha_i\tau. \tag{5}$$

Recalling that $m_C = m_D = 4\,\mathrm{lb}/g$ $g = 32.2\,\mathrm{ft/s^2}$, $r = 0.75\,\mathrm{ft}$, $m_A = m_B = 150\,\mathrm{lb}/g$, $R = 3.5\,\mathrm{ft}$, $\alpha_i = 5.0\,\mathrm{rad/s^2}$, and $\tau = 10\,\mathrm{s}$, we can evaluate the angular speed $|\vec{\omega}_{s2}| = |\omega_{s2}|$ to obtain

$$\boxed{\left|\vec{\omega}_{s2}\right| = 0.06122\,\mathrm{rad/s}.}$$