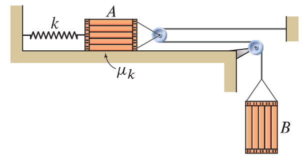
Problem 4.77

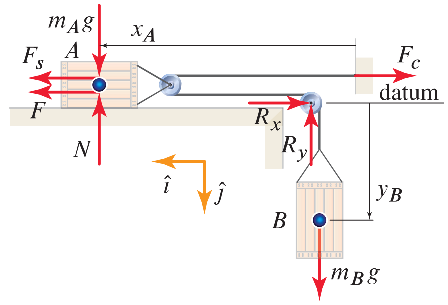
Crates A and B of mass 50 kg and 75 kg, respectively, are released from rest. The linear elastic spring has stiffness $k = 500\,\mathrm{N/m}$. Neglect the mass of the pulleys and cables and neglect friction in the pulley bearings.

If $\mu_k = 0.25$ and the spring is initially stretched 1 m, how far does A slide before the system momentarily comes to rest?



Solution

We model A and B as a system of particles subject to the spring force F_s, the weights of A and B, $m_A g$ and $m_B g$, respectively, the reaction with the ground N, the friction force F, the reactions R_x and R_y at the pulley from which B hangs, and the tension F_c in the cord. We have included the pulleys and the cord as elements of the system because their inertia is negligible. We observe that R_x, R_y, and F_c do no work since their respective points of application are fixed. We denote by ① and ② the positions of the system at release and when it comes momentarily to rest, respectively. The subscripts 1 and 2 denote quantities at ① and ②, respectively.



Balance Principles. Applying the work-energy principle, we have

$$T_1 + V_1 + (U_{1\text{-}2})_{\text{nc}} = T_2 + V_2, \tag{1}$$

where V is the potential energy of the system, and where, denoting by v_A and v_B the speeds of A and B, respectively, we have

$$T_1 = \tfrac{1}{2} m_A v_{A1}^2 + \tfrac{1}{2} m_B v_{B1}^2 \quad \text{and} \quad T_2 = \tfrac{1}{2} m_A v_{A2}^2 + \tfrac{1}{2} m_B v_{B2}^2. \tag{2}$$

Force Laws. Given the choice of datum, and denoting by δ the stretch of the spring and by h the drop in elevation of B from ① to ②, we have

$$V_1 = \tfrac{1}{2} k \delta_1^2 - m_B g y_{B1} \quad \text{and} \quad V_2 = \tfrac{1}{2} k \delta_2^2 - m_B g (y_{B1} + h). \tag{3}$$

To determine the work of the friction force, we observe that, since A does not move vertically, equilibrium in the vertical direction demands that $N = m_A g$. Therefore, $F = \mu_k m_A g$ and its work between ① and ② is

$$U_{1\text{-}2} = -\mu_k m_A g d. \tag{4}$$

Kinematic Equations. The stretch of the spring in ① is a given of the problem. For ② we have

$$\delta_2 = \delta_1 + d. \tag{5}$$

Also, denoting by L the length of the cable in the pulley system, we have

$$L = 2x_A + y_B = \text{constant} \quad \Rightarrow \quad 2(x_{A1} - x_{A2}) = y_{B2} - y_{B1} \quad \Rightarrow \quad h = 2d. \tag{6}$$

The system is at rest in ① and ② so

$$v_{A1} = 0, \quad v_{B1} = 0, \quad v_{A2} = 0, \quad v_{B2} = 0. \tag{7}$$

Computation. Substituting Eqs. (7) into Eqs. (2), we have

$$T_1 = 0 \quad \text{and} \quad T_2 = 0. \tag{8}$$

Substituting Eq. (5) and the last of Eqs. (6) into Eq. (3), we have

$$V_1 = \tfrac{1}{2}k\delta_1^2 - m_B g y_{B1} \quad \text{and} \quad V_2 = \tfrac{1}{2}k(\delta_1 + d)^2 - m_B g(y_{B1} + 2d). \tag{9}$$

Substituting Eqs. (8), (9), and (4) into Eq. (1), we have

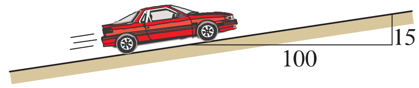
$$\tfrac{1}{2}k\delta_1^2 - m_B g y_{B1} - \mu_k m_A g d = \tfrac{1}{2}k(\delta_1 + d)^2 - m_B g(y_{B1} + 2d)$$
$$\Rightarrow \quad d = (2g/k)(2m_B - \mu_k m_A) - 2\delta_1 \tag{10}$$

Recalling that $m_A = 50\,\text{kg}$, $m_B = 75\,\text{kg}$, $\mu_k = 0.25$, $k = 500\,\text{N/m}$, $g = 9.81\,\text{m/s}^2$, and $\delta_1 = 1\,\text{m}$, we can evaluate d to obtain

$$\boxed{d = 3.396\,\text{m}.}$$

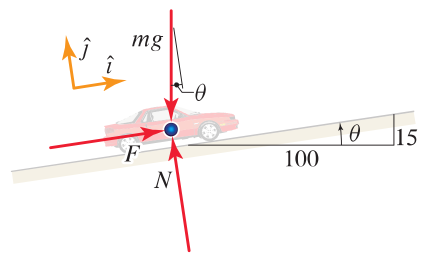
Problem 4.105

A 1500 kg car is traveling up the slope shown at a constant speed. Knowing that the maximum power output of the car is 160 hp, at what speed can the car travel if air resistance is negligible? Also, knowing that 1 L of regular gasoline provides 34.8 MJ of energy, how many liters of gasoline will be required in 1 h if the engine has an efficiency $\epsilon = 0.20$?



Solution

We model the car as a particle subject to its own weight mg, the normal reaction with the road N, and the propulsive force F.



Balance Principles. Applying Newton's second law in the x direction, we have

$$\sum F_x: \quad F - mg\sin\theta = ma_x, \tag{1}$$

where a_x is the x component of the acceleration of the car.

Force Laws. All forces are accounted for on the FBD.

Kinematic Equations. The car is assumed to move at a constant speed in the x direction, so

$$a_x = 0. \tag{2}$$

Computation. Substituting Eq. (2) into Eq. (1), we have that

$$F = mg\sin\theta. \tag{3}$$

Given the power P of the engine and recalling that $P = Fv$, we have

$$P = Fv \quad \Rightarrow \quad v = \frac{P}{mg\sin\theta} \quad \Rightarrow \quad \boxed{v = 54.66\,\text{m/s} = 196.8\,\text{km/h},} \tag{4}$$

where we have used the data $P = 160\,\text{hp} = (745.7)(160)\,\text{W}$, $m = 1500\,\text{kg}$, $\theta = \tan^{-1}(15/100)$, and $g = 9.81\,\text{m/s}^2$.

We denote one hours of time by τ. Also, we denote by $\gamma = 34.8\,\text{MJ/L}$ the amount of energy per liter that gasoline provides. To determine the volume of gasoline needed during a time interval equal to τ, we first need to determine the time rate of energy P_i that needs to be provided to the engine. Denoting the engine's efficiency by ϵ, and applying the definition of efficiency, we have

$$P_i = P/\epsilon. \tag{5}$$

Hence, the energy E needed in an hour is

$$E = P_i\tau \quad \Rightarrow \quad E = P\tau/\epsilon, \tag{6}$$

and the volume of gasoline V_{gasoline} needed in one hour is

$$V_{\text{gasoline}} = E/\gamma \quad \Rightarrow \quad V_{\text{gasoline}} = P\tau/(\epsilon\gamma). \tag{7}$$

Recalling that $P = 160\,\text{hp} = (745.7)(160)\,\text{W}$, $\epsilon = 0.20$, $\tau = 1\,\text{h} = 3600\,\text{s}$, and $\gamma = 34.8\,\text{MJ/L} = (34.8)10^6\,\text{J/L}$, we van evaluate V_{gasoline} to obtain

$$\boxed{V_{\text{gasoline}} = 61.71\,\text{L}.}$$