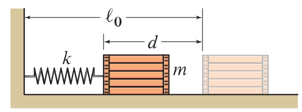
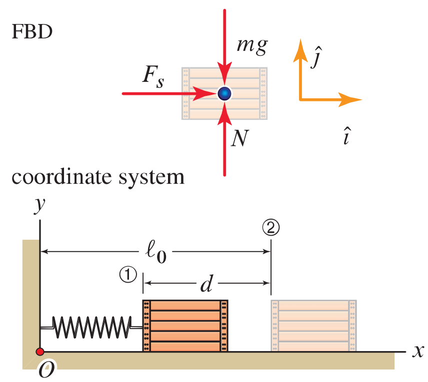
Problem 4.8

The crate of mass m is pushed to the left until the linear elastic spring of constant k has compressed a distance d from its unstretched length ℓ_0. The crate is released from rest, and friction between the crate and the horizontal surface is negligible.

Using the work-energy principle, determine the speed of the crate at the instant the spring becomes uncompressed.



Solution



Referring to the bottom part of the figure at the right, we denote by ① and ② the positions at which the crate is first released and at which the spring becomes uncompressed, respectively. As shown in the FBD (upper part of the figure), between ① and ②, we model the crate as a particle subject to its own weight mg, the spring force F_s, and the normal reaction N with the ground. We will use subscripts 1 and 2 to denote quantities at ① and ②, respectively.

Balance Principles. Applying the work-energy principle between ① and ②, we have

$$T_1 + U_{1\text{-}2} = T_2, \tag{1}$$

where $U_{1\text{-}2}$ is the work done on the crate from ① to ②, and where, denoting by v the speed of the crate,

$$T_1 = \tfrac{1}{2}mv_1^2 \quad \text{and} \quad T_2 = \tfrac{1}{2}mv_2^2. \tag{2}$$

Force Laws. Letting $\vec{F}_t$ be the total force acting on the crate, the work of $\vec{F}_t$ is

$$U_{1\text{-}2} = \int_{\mathcal{L}_{1\text{-}2}} \vec{F}_t \cdot d\vec{r} \quad \Rightarrow \quad U_{1\text{-}2} = \int_{x_1}^{x_2} k(\ell_0 - x)\,dx = \tfrac{1}{2}k\big[(\ell_0 - x_1)^2 - (\ell_0 - x_2)^2\big], \tag{3}$$

where, using the FBD and the chosen coordinate system, $\vec{F}_t = F_s\,\hat{\imath} + (N - mg)\,\hat{\jmath}$, $F_s = k(\ell_0 - x)$, and $d\vec{r} = dx\,\hat{\imath}$ since the crate moves only in the x direction.

Kinematic Equations. Using chosen coordinate system and the definition of ① and ②, we have

$$x_1 = \ell_0 - d, \quad x_2 = \ell_0, \quad v_1 = 0. \tag{4}$$

Computation. Substituting Eqs. (2) and the last of Eqs. (4) into Eq. (1), we have

$$U_{1\text{-}2} = \tfrac{1}{2}mv_2^2 \quad \Rightarrow \quad v_2 = \sqrt{2U_{1\text{-}2}/m}. \tag{5}$$

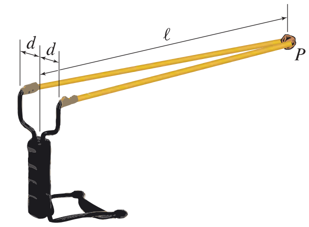
Substituting the first two of Eqs. (4) into the last of Eqs. (3), and then substituting the result into the last of Eqs. (5), we have

$$\boxed{v_2 = d\sqrt{k/m}.}$$

Problem 4.41

Each of the two rubber tubes of the wrist rocket has an unstretched length $L_0 = 1$ ft. They are symmetrically pulled back so that $\ell = 3$ ft and then are released from rest. The pellet P that is launched by the wrist rocket weighs 0.145 oz. Neglect the mass of the rubber tubes and any change in height of the pellet while it is in contact with the rubber tubes. Finally, let $d = 1.5$ in.

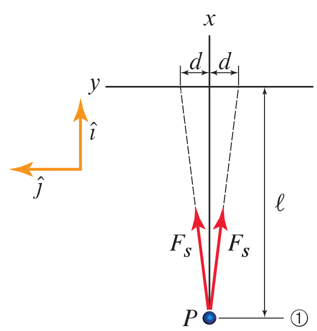
If the desired launch speed of the pellet P is 100 mph, determine the required stiffness of each rubber tube.



Solution

NOTE: The quantity $d = 1.5$ in.

Let ① be the position at which the rubber tubes are released and ② be the position at which the rubber tubes become unstretched (the rubber tubes do not propel the pellet beyond this point). Neglecting the vertical motion of the pellet, we assume that between ① and ②, the pellet is a particle subject only to the forces F_s due to the rubber tubes. We will use subscripts 1 and 2 to denote quantities at ① and ②, respectively. We observe that the only forces present in the FBD are conservative so the work-energy principle can be written as the conservation of energy statement.



Balance Principles. Applying the work-energy principle between ① and ②, we have

$$T_1 + V_1 = T_2 + V_2, \tag{1}$$

where V is the potential energy of the pellet and where, denoting by v the speed of the pellet,

$$T_1 = \tfrac{1}{2}mv_1^2 \quad \text{and} \quad T_2 = \tfrac{1}{2}mv_2^2. \tag{2}$$

Force Laws. Denoting by δ the stretch in each rubber tube, and recalling that we have a rubber tube on either side of the pellet, V_1 and V_2 are

$$V_1 = 2\left(\tfrac{1}{2}k\delta_1^2\right) \quad \text{and} \quad V_2 = 2\left(\tfrac{1}{2}k\delta_2^2\right). \tag{3}$$

Kinematic Equations. Recalling that the pellet is released from rest, we have

$$v_1 = 0, \quad v_2 = 100\,\text{mph} = 100\tfrac{5280}{3600}\,\text{ft/s}. \tag{4}$$

Also, we have

$$\delta_1 = \sqrt{\ell^2 + d^2} - L_0 \quad \text{and} \quad \delta_2 = 0. \tag{5}$$

Computation. Substituting Eqs. (5) into Eqs. (3), we have

$$V_1 = k\left(\sqrt{\ell^2 + d^2} - L_0\right)^2 \quad \text{and} \quad V_2 = 0. \tag{6}$$

Substituting this result, Eqs. (2) and the first of Eqs. (4) into Eq. (1), we have

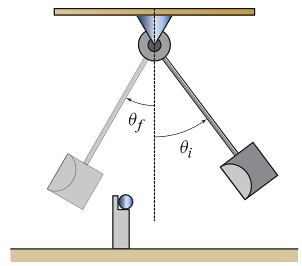
$$k\left(\sqrt{\ell^2 + d^2} - L_0\right)^2 = \tfrac{1}{2}mv_2^2 \quad \Rightarrow \quad k = \frac{mv_2^2}{2\left(\sqrt{\ell^2 + d^2} - L_0\right)^2}. \tag{7}$$

Using the second of Eqs. (4) and recalling that $m = 0.145\,\text{oz}/g = \frac{0.145}{16}\,\text{lb}/g$, $g = 32.2\,\text{ft/s}^2$, $\ell = 3\,\text{ft}$, $d = 1.5\,\text{in.} = \frac{1.5}{12}\,\text{ft}$, and $L_0 = 1\,\text{ft}$, we can evaluate k to obtain

$$\boxed{k = 0.7548\,\text{lb/ft}.}$$

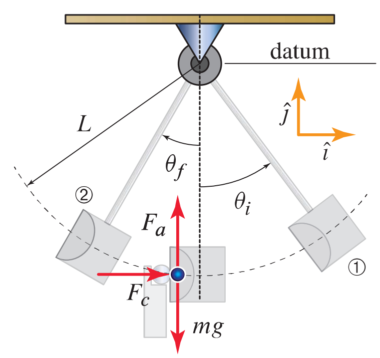
Problem 4.59

The resistance of a material to fracture is assessed with a fracture test. One such test is the *Charpy impact test*, in which the fracture toughness is assessed by measuring the energy required to break a specimen of a specified geometry. This is done by releasing a heavy pendulum from rest at an angle θ_i and by measuring the maximum swing angle θ_f reached by the pendulum after the specimen is broken. Suppose that in an experiment $\theta_i = 45°$, $\theta_f = 23°$, the weight of the pendulum's bob is 3 lb, and the length of the pendulum is 3 ft. Neglecting the mass of any other component of the testing apparatus, assuming that the pendulum's pivot is frictionless, and treating the pendulum's bob as a particle, determine the fracture energy of the specimen tested. Assume that the fracture energy is the energy required to break the specimen.



Solution

We model the bob as a particle subject to its own weight mg, the tension in the pendulum arm F_a, and the contact force with the specimen F_c. Clearly, this force is considered equal to zero when the bob is not in contact with the specimen. We denote by ① the position at which the bob is released. We denote by ② the position at which the bob stops. We use subscripts 1 and 2 to denote quantities at ① and ②, respectively. We observe that F_a does no work because the arm can be modeled as inextensible. Therefore, all of the work done between ① and ② is due to the weight mg, which is conservative, and the contact force with the specimen. In absolute value, the latter work corresponds to the energy required to break the specimen.



Balance Principles. Applying the work-energy principle between ① and ②, we have

$$T_1 + V_1 + (U_{1\text{-}2})_{\text{nc}} = T_2 + V_2, \tag{1}$$

where V is the potential energy of the bob, $(U_{1\text{-}2})_{\text{nc}}$ is the energy required to break the specimen, and where, denoting by v the speed of the bob,

$$T_1 = \tfrac{1}{2}mv_1^2 \quad \text{and} \quad T_2 = \tfrac{1}{2}mv_2^2. \tag{2}$$

Force Laws. We do not provide an expression for $(U_{1\text{-}2})_{\text{nc}}$ since this is the quantity we want to determine. As for V, due to the choice of datum and denoting by L the length of the arm of the pendulum, we have

$$V_1 = -mgL\cos\theta_i \quad \text{and} \quad V_2 = -mgL\cos\theta_f. \tag{3}$$

Kinematic Equations. The bob is released from rest in ① and comes to a stop in ②. So

$$v_1 = 0 \quad \text{and} \quad v_2 = 0. \tag{4}$$

Computation. Substituting Eqs. (2)–(4) into Eq. (1), we have

$$-mgL\cos\theta_i + (U_{1\text{-}2})_{\text{nc}} = -mgL\cos\theta_f \quad \Rightarrow \quad (U_{1\text{-}2})_{\text{nc}} = mgL(\cos\theta_i - \cos\theta_f). \tag{5}$$

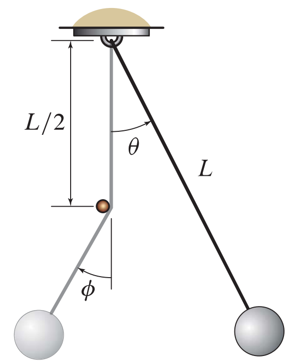
The quantity $(U_{1\text{-}2})_{\text{nc}}$ is the work done on the pendulum bob by the specimen. Hence the energy required to break the specimen is the negative of $(U_{1\text{-}2})_{\text{nc}}$. Keeping this in mind, and recalling that $m = 3\,\text{lb}/g$, $g = 32.2\,\text{ft/s}^2$, $L = 3\,\text{ft}$, $\theta_i = 45°$, and $\theta_f = 23°$, we can evaluate $(U_{1\text{-}2})_{\text{nc}}$ to obtain

Energy required to break the specimen $= 1.921$ ft·lb.

Problem 4.60

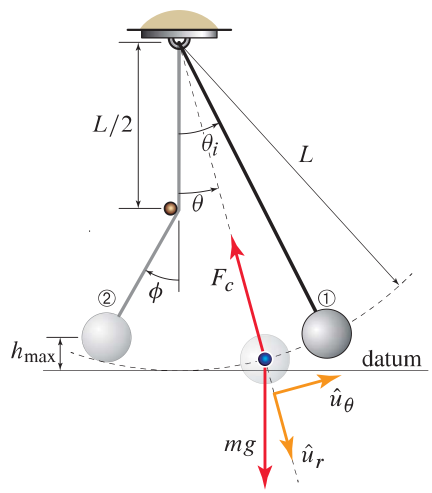
A pendulum with mass $m = 1.4\,\text{kg}$ and length $L = 1.75\,\text{m}$ is released from rest at an angle θ_i. Once the pendulum has swung to the vertical position (i.e., $\theta = 0$), its cord runs into a small fixed obstacle. In solving this problem, neglect the size of the obstacle, model the pendulum's bob as a particle, model the pendulum's cord as massless and inextensible, and let gravity and the tension in the cord be the only relevant forces.

What is the maximum height, measured from its lowest point, reached by the pendulum if $\theta_i = 20°$?



Solution

We denote by ① the position of the pendulum at release, and by ② the position at which the pendulum bob achieves the maximum height (relative to the bottom of its trajectory). We denote quantities at ① and ② by subscripts 1 and 2, respectively. Between ① and ②, we model the pendulum bob as a particle subject to its own weight mg and the tension in the cord F_c. Since the cord is assumed to be inextensible, F_c does no work. Hence, the only force doing work is mg, which is conservative.



Balance Principles. Applying the work-energy principle, we have

$$T_1 + V_1 = T_2 + V_2, \tag{1}$$

where V is the potential energy of the system, and where, denoting by v the speed of the bob,

$$T_1 = \tfrac{1}{2}mv_1^2 \quad \text{and} \quad T_2 = \tfrac{1}{2}mv_2^2. \tag{2}$$

Force Laws. Due to our choice of datum, we have

$$V_1 = mgL(1 - \cos\theta_i) \quad \text{and} \quad V_2 = mgh_{\text{max}}. \tag{3}$$

Kinematic Equations. The pendulum is released from rest and it comes to a stop in ②. Hence,

$$v_1 = 0 \quad \text{and} \quad v_2 = 0. \tag{4}$$

Computation. Substituting Eqs. (2)–(4) into Eq. (1), we have

$$mgL(1 - \cos\theta_i) = mgh_{\text{max}} \quad \Rightarrow \quad h_{\text{max}} = L(1 - \cos\theta_i). \tag{5}$$

Recalling that $L = 1.75\,\text{m}$ and $\theta_i = 20°$, we can evaluate h_{max} to obtain

$$\boxed{h_{\text{max}} = 0.1055\,\text{m}.}$$