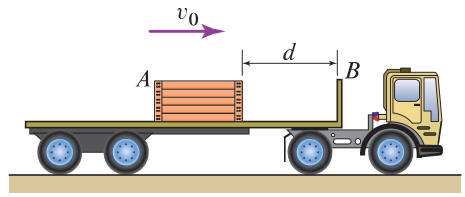
Problem 3.20

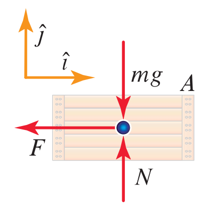
The truck shown is traveling at $v_0 = 60$ mph when the driver applies the brakes to come to a stop. The deceleration of the truck is constant, and the truck comes to a complete stop after braking for a distance of 350 ft. Treat the crate as a particle so that tipping can be neglected.

Determine the minimum coefficient of static friction between the crate A and the truck so that the crate does *not* slide relative to the truck.



Solution

Using the information provided we first determine the (constant) acceleration of the truck. This acceleration is also that of the crate (the crate does not slip relative to the truck) and can be related to the needed friction force via Newton's second law. We model the crate as a particle subject only to its own weight mg, the normal reaction N between the crate and the truck bed, and the friction force F. Since we need to compute the minimum coefficient of static friction $(\mu_s)_{\min}$ required for the motion, we assume that the crate is in an impending slip condition.



Balance Principles. Applying Newton's second law, we have

$$\sum F_x: \quad -F = ma_{Ax}, \tag{1}$$

$$\sum F_y: \quad N - mg = ma_{Ay}, \tag{2}$$

a_{Ax} and a_{Ay} are the horizontal and vertical components of the acceleration of A.

Force Laws. Since the crate is assumed to be in an impending slip condition, we have

$$F = (\mu_s)_{\min} N. \tag{3}$$

Kinematic Equations. In the horizontal direction, the crate goes from the speed v_0 to 0 with a constant acceleration a_{Ax} over the braking distance d_b. Therefore, using a coordinate system with origin at the location where the brakes are first applied, and using constant acceleration equations we have

$$0 - v_0^2 = 2a_{Ax}(d_b - 0) \quad \Rightarrow \quad a_{Ax} = -\frac{v_0^2}{2d_b}. \tag{4}$$

The crate will not move in the vertical direction so $a_{Ay} = 0$.

Computation. Substituting Eq. (3) and the last of Eqs. (4) into Eq. (1), and $a_{Ay} = 0$ into Eq. (2) gives

$$-(\mu_s)_{\min} N = -m\frac{v_0^2}{2d_b} \quad \text{and} \quad N - mg = 0, \tag{5}$$

which is a system of two equations in the two unknowns N and $(\mu_s)_{\min}$ with solution

$$N = mg \quad \text{and} \quad (\mu_s)_{\min} = \frac{v_0^2}{2d_b g}. \tag{6}$$

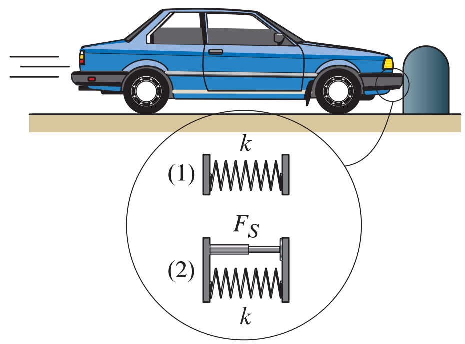
Recalling that $v_0 = 60\,\text{mph} = 60\frac{5280}{3600}\,\text{ft/s}$, $d_b = 350\,\text{ft}$, and $g = 32.2\,\text{ft/s}^2$, we can evaluate the second of Eqs. (6) to obtain

$$\boxed{(\mu_s)_{\min} = 0.3436.}$$

Problem 3.30

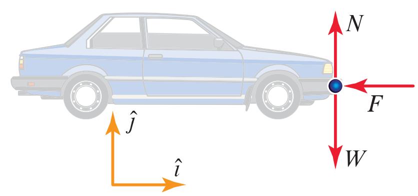
Car bumpers are designed to limit the extent of damage to the car in the case of low-velocity collisions. Consider a 3300 lb passenger car impacting a concrete barrier while traveling at a speed of 4.0 mph. Model the car as a particle, and consider two types of bumpers: (1) a simple linear spring with constant k and (2) a linear spring of constant k in parallel with a shock-absorbing unit generating a nearly constant force of 700 lb over 0.25 ft.

If the bumper is of type 1 and if $k = 6500$ lb/ft, find the spring compression necessary to stop the car.



Solution

In the FBD at the right, we model the car as a particle subject only to its weight W, the normal reaction with the ground N, and the force F of the bumper. We assume that the motion of the car is only in the x direction. The compression of the bumper necessary to stop the car can be computed by applying Newton's second law to determine the stopping distance of the car.



Balance Principles. Applying Newton's second law, we have

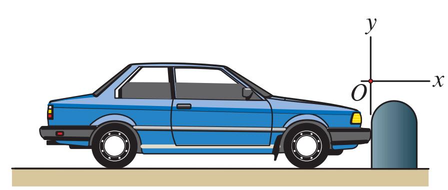
$$\sum F_x: \quad -F = \frac{W}{g} a_x, \tag{1}$$

$$\sum F_y: \quad N - W = \frac{W}{g} a_y, \tag{2}$$

where g is the acceleration due to gravity so that W/g is the mass of the car, and where a_x and a_y are the x and y components of the acceleration of the car.

Force Laws. We model the bumper as a linear spring. Choosing the origin O of the xy coordinate system to coincide with the position of the car when the bumper first touches the barrier,

$$F = kx. \tag{3}$$



Kinematic Equations. We have

$$a_x = \ddot{x} \quad \text{and} \quad a_y = 0, \tag{4}$$

where the first of Eqs. (4) relates the horizontal acceleration of the car to the car's x coordinate, and where the second of Eqs. (4) is due to the assumption that the car does not move in the vertical direction.

Computation. Substituting Eqs. (3) and (4) into Eqs. (1) and (2) we have

$$-kx = \frac{W}{g}\ddot{x} \quad \text{and} \quad N - W = 0. \tag{5}$$

This is a system of two equations in the two unknowns $\ddot{x}$ and N whose solution is

$$\ddot{x} = -\frac{kg}{W}x \quad \text{and} \quad N = W. \tag{6}$$

Using the chain rule, $\ddot{x} = \dot{x}\,d\dot{x}/dx$. Therefore, the first of Eqs. (6) gives

$$\dot{x}\frac{d\dot{x}}{dx} = -\frac{kg}{W}x \quad \Rightarrow \quad \dot{x}\,d\dot{x} = -\frac{kg}{W}x\,dx, \tag{7}$$

where the last of Eqs. (7) has been obtained by separating the variables x and $\dot{x}$. Denoting by x_{stop} the stopping position of the car, recalling that $\dot{x} = \dot{x}_0 = 4\,\text{mph}$ for $x = 0$, and that $\dot{x} = 0$ for $x = x_{\text{stop}}$, we can integrate the last of Eqs. (7) as follows:

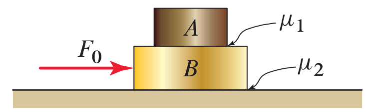
$$\int_{\dot{x}_0}^{0} \dot{x}\,d\dot{x} = \int_0^{x_{\text{stop}}} -\frac{kg}{W}x\,dx \quad \Rightarrow \quad -\tfrac{1}{2}\dot{x}_0^2 = -\frac{kg}{2W}x_{\text{stop}}^2 \quad \Rightarrow \quad x_{\text{stop}} = \sqrt{\frac{\dot{x}_0^2 W}{kg}}, \tag{8}$$

where we have discarded the root $x_{\text{stop}} = -\sqrt{\dot{x}_0^2 W/(kg)}$ because the car is initially moving in the positive x direction. We now observe that x_{stop} coincides with the compression of the bumper required for the car to stop. Therefore, recalling that $\dot{x}_0 = 4\,\text{mph} = 4\frac{5280}{3600}\,\text{ft/s}$, $W = 3300\,\text{lb}$, $g = 32.2\,\text{ft/s}^2$, and $k = 6500\,\text{lb/ft}$, we can evaluate the last of Eqs. (8) to obtain

Spring compression $= 0.7367\,\text{ft}$.

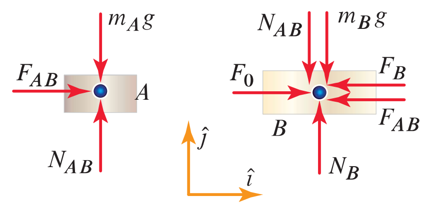
Problem 3.115

A force F_0 of 400 lb is applied to block B. Letting the weights of A and B be 55 and 73 lb, respectively, and letting the static *and* kinetic friction coefficients between blocks A and B be $\mu_1 = 0.25$, and the static *and* kinetic friction coefficients between block B and the ground be $\mu_2 = 0.45$, determine the accelerations of both blocks.



Solution

Referring to the figure at the right, we sketch individual FBDs for A and B, each modeled as a particle. We model A as subject to its own weight $m_A g$, the normal force N_{AB} between A and B, and the friction force F_{AB}. We use the working assumption that A slides to the left relative to B. We model B as subject to the force F_0, the forces F_{AB} and N_{AB} with their direction reversed to satisfy Newton's third law, as well as its own weight $m_B g$, the normal force N_B with the ground, and the friction force F_B. We use the working assumption that B slides over the ground. For the stated working assumptions to be correct we must verify that the solution satisfies the conditions $0 < a_{Ax} < a_{Bx}$.



Balance Principles. Applying Newton's second law to each particle, we have

$$\left(\sum F_x\right)_A: \quad F_{AB} = m_A a_{Ax}, \qquad \left(\sum F_y\right)_A: \quad N_{AB} - m_A g = m_A a_{Ay}, \tag{1}$$

$$\left(\sum F_x\right)_B: \quad F_0 - F_{AB} - F_B = m_B a_{Bx}, \qquad \left(\sum F_y\right)_B: \quad N_B - m_B g - N_{AB} = m_B a_{By}, \tag{2}$$

where a_{Ax} and a_{Ay} are the x and y components of the acceleration of A, and a_{Bx} and a_{By} are the x and y components of the acceleration of B.

Force Laws. Because of our working assumptions, we have

$$F_{AB} = \mu_1 N_{AB} \quad \text{and} \quad F_B = \mu_2 N_B. \tag{3}$$

Kinematic Equations. A and B do not move in the vertical direction. Hence,

$$a_{Ay} = 0 \quad \text{and} \quad a_{By} = 0. \tag{4}$$

Computation. Substituting the first of Eqs. (3) and Eqs. (4) into Eqs. (1) and (2) we have

$$\mu_1 N_{AB} = m_A a_{Ax}, \quad N_{AB} - m_A g = 0, \quad F_0 - \mu_1 N_{AB} - \mu_2 N_B = 0, \quad N_B - m_B g - N_{AB} = 0. \tag{5}$$

This is a system of four equations in the four unknowns N_{AB}, N_B, a_{Ax}, and a_{Bx} whose solution is

$$N_{AB} = m_A g, \quad N_B = (m_A + m_B)g, \quad a_{Ax} = \mu_1 g, \quad a_{Bx} = \frac{F_0}{m_B} - \left[\mu_2 + (\mu_1 + \mu_2)\frac{m_A}{m_B}\right]g. \tag{6}$$

Recalling that $F_0 = 400\,\text{lb}$, $m_A = 55\,\text{lb}/g$, $m_B = 73\,\text{lb}/g$, $g = 32.2\,\text{ft/s}^2$, $\mu_1 = 0.25$, and $\mu_2 = 0.45$, we can evaluate a_{Ax} and a_{Bx} to obtain $a_{Ax} = 8.050\,\text{ft/s}^2$ and $a_{Bx} = 145.0\,\text{ft/s}^2$. Since $0 < a_{Ax} < a_{Bx}$, our solution is indeed the true solution and, expressing our answer in vector form, we have,

$$\vec{a} = 8.050\,\hat{\imath}\ \text{ft/s}^2 \quad \text{and} \quad \vec{a}_B = 145.0\,\hat{\imath}\ \text{ft/s}^2.$$