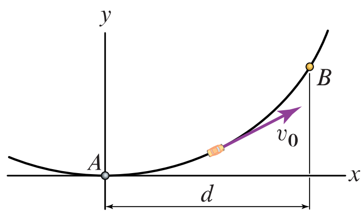
Problem 2.153

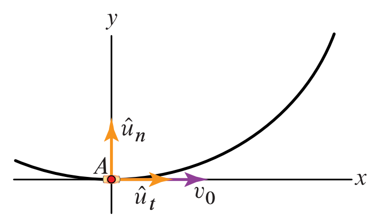
The portion of a race track between points A (corresponding to $x = 0$) and B is part of a parabolic curve described by the equation $y = \kappa x^2$, where κ is a constant. Let g denote the acceleration due to gravity.

Determine κ such that a car driving at constant speed $v_0 = 180$ mph experiences at A an acceleration with magnitude equal to $1.5g$.



Solution

Referring to the figure at the right, at point A the tangent and normal directions to the trajectory coincide with the x and the y axes, respectively. The expression of the acceleration in normal-tangential components is



$$\vec{a} = \dot{v}\,\hat{u}_t + \frac{v^2}{\rho}\,\hat{u}_n, \tag{1}$$

where v is the speed and ρ is the radius of curvature. Since the speed is constant and equal to v_0, we have that $\dot{v} = 0$ and Eq. (1) reduces to

$$\vec{a} = \frac{v_0^2}{\rho}\,\hat{u}_n. \tag{2}$$

We can determine ρ using Eq. (2.59) on p. 93 of the textbook, namely,

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{\left|d^2y/dx^2\right|} \quad\Rightarrow\quad \rho = \frac{\left[1 + (2\kappa x)^2\right]^{3/2}}{2\kappa}, \tag{3}$$

where, recalling that $y = \kappa x^2$, we have used the fact that $dy/dx = 2\kappa x$ and $d^2y/dx^2 = 2\kappa$. Substituting the last of Eqs. (3) into Eq. (2) gives

$$\vec{a} = \frac{2\kappa v_0^2}{\left[1 + (2\kappa x)^2\right]^{3/2}}\,\hat{u}_n. \tag{4}$$

Recalling that $|\vec{a}| = 1.5g$ for $x = 0$, from Eq. (4) we have

$$2\kappa v_0^2 = 1.5g = \tfrac{3}{2}g \quad\Rightarrow\quad \kappa = \frac{3g}{4v_0^2}. \tag{5}$$

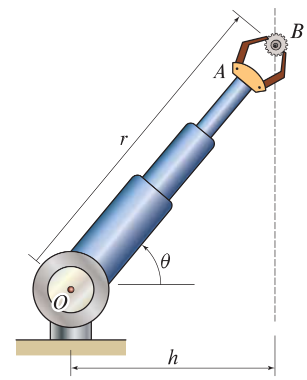
Recalling that $g = 32.2\,\text{ft/s}^2$ and $v_0 = 180\,\text{mph} = 180(5280/3600)\,\text{ft/s}$, we can evaluate the last of Eqs. (5) to obtain

$$\kappa = 0.3465\times 10^{-3}\,\text{ft}^{-1}.$$

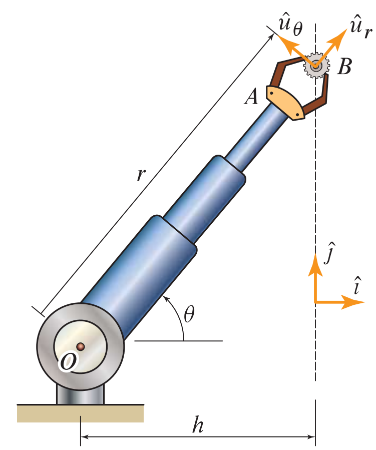
Problem 2.206

As a part of an assembly process, the end effector at A on the robotic arm needs to move the gear at B along the vertical line shown with some known velocity v_0 and acceleration a_0. Arm OA can vary its length by telescoping via internal actuators, and a motor at O allows it to pivot in the vertical plane.

When $\theta = 50°$, it is required that $v_0 = 8\,\text{ft/s}$ (down) and that it be slowing down at $a_0 = 2\,\text{ft/s}^2$. Using $h = 4\,\text{ft}$, determine, at this instant, the values for $\ddot{r}$ (the extensional acceleration) and $\ddot{\theta}$ (the angular acceleration).



Solution



Referring to the figure at the right, the length of the arm as a function of θ is

$$r = h/\cos\theta. \tag{1}$$

The velocity of B can be expressed in both the Cartesian and polar component systems shown. Since B moves downward, this gives

$$\vec{v}_B = -v_0\,\hat{\jmath} = \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta. \tag{2}$$

We note that

$$\hat{\jmath} = \sin\theta\,\hat{u}_r + \cos\theta\,\hat{u}_\theta, \tag{3}$$

so that Eq. (2) can be written as $-v_0(\sin\theta\,\hat{u}_r + \cos\theta\,\hat{u}_\theta) = \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta$, which implies $\dot{r} = -v_0\sin\theta$ and $r\dot{\theta} = -v_0\cos\theta$, i.e.,

$$\dot{r} = -v_0\sin\theta \quad \text{and} \quad \dot{\theta} = -\frac{v_0\cos^2\theta}{h}, \tag{4}$$

where we have used Eq. (1). Since B is slowing down (in its downward motion), the acceleration of B, using both component systems, is

$$\vec{a}_B = a_0\,\hat{\jmath} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{u}_\theta$$
$$\Rightarrow \quad a_0(\sin\theta\,\hat{u}_r + \cos\theta\,\hat{u}_\theta) = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{u}_\theta, \tag{5}$$

where we have used Eq. (3). Equating components, we obtain

$$\ddot{r} - r\dot{\theta}^2 = a_0\sin\theta \quad \text{and} \quad r\ddot{\theta} + 2\dot{r}\dot{\theta} = a_0\cos\theta. \tag{6}$$

Using the results from Eqs. (1) and (4), Eqs. (6) give

$$\ddot{r} = a_0\sin\theta + \frac{h}{\cos\theta}\left(-\frac{v_0\cos^2\theta}{h}\right)^2 \quad \Rightarrow \quad \ddot{r} = a_0\sin\theta + \frac{v_0^2\cos^3\theta}{h} \quad \Rightarrow \quad \boxed{\ddot{r} = 5.781\,\text{ft/s}^2,}$$

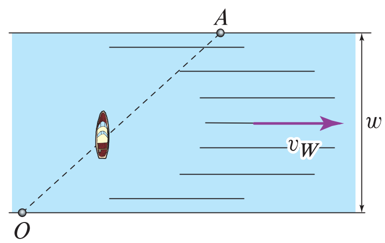
and

$$\ddot{\theta} = \frac{a_0\cos^2\theta}{h} - 2(-v_0\sin\theta)\left(-\frac{v_0\cos^2\theta}{h}\right)\left(\frac{\cos\theta}{h}\right) \quad \Rightarrow \quad \ddot{\theta} = \frac{a_0\cos^2\theta}{h} - \frac{2v_0^2\cos^3\theta\sin\theta}{h^2}$$
$$\Rightarrow \quad \boxed{\ddot{\theta} = -1.421\,\text{rad/s}^2,}$$

where we have used the following numerical data $\theta = 50°$, $v_0 = 8\,\text{ft/s}$, $a_0 = 2\,\text{ft/s}^2$, and $h = 4\,\text{ft}$.

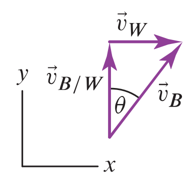
Problem 2.227

A remote controlled boat, capable of a maximum speed of 10 ft/s in still water, is made to cross a stream with a width $w = 35$ ft that is flowing with a speed $v_W = 7$ ft/s. If the boat starts from point O and keeps its orientation parallel to the cross-stream direction, find the location of point A at which the boat reaches the other bank while moving at its maximum speed. Furthermore, determine how much time the crossing requires.



Solution

Using a Cartesian coordinate system with its origin at O, as the boat is crossing the stream, its velocity $\vec{v}_B$ can be written as



$$\vec{v}_B = \vec{v}_W + \vec{v}_{B/W} = (7\,\hat{\imath} + 10\,\hat{\jmath})\ \text{ft/s},$$

where $\vec{v}_W = 7\,\hat{\imath}$ ft/s is the velocity of the water and $\vec{v}_{B/W} = 10\,\hat{\jmath}$ ft/s is the velocity of the boat relative to the water. Using the the y component of velocity, the time of crossing is

$$t = \frac{w}{v_{By}} \quad \Rightarrow \quad \boxed{t = 3.500\ \text{s},} \tag{1}$$

where $w = 35$ ft. Since the x component of velocity is constant, using the crossing time in Eq. (1), we can calculate the downstream position of A as

$$x = v_{Bx} t \quad \Rightarrow \quad \boxed{x = 24.50\ \text{ft}.}$$