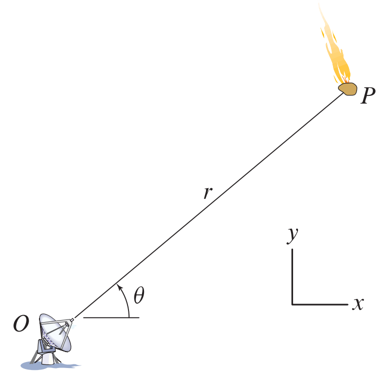
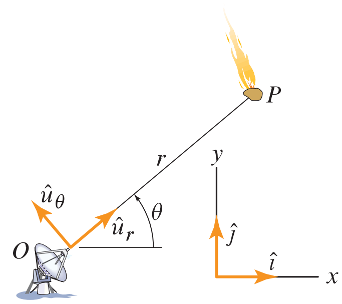
Problem 2.127

The radar station at O is tracking the meteor P as it moves through the atmosphere. At the instant shown, the station measures the following data for the motion of the meteor: $r = 21{,}000$ ft, $\theta = 40°$, $\dot{r} = -22{,}440$ ft/s, and $\dot{\theta} = -2.935$ rad/s. Use Eq. (2.48) to determine the magnitude and direction (relative to the xy coordinate system shown) of the velocity vector at this instant.



Solution



Referring to the figure at the right, the unit vector $\hat{u}_r$ always points toward P. The unit vector $\hat{u}_\theta$ is perpendicular to $\hat{u}_r$ and points in the direction of increasing θ. Then, letting r denote the distance between P and the fixed point O, we have that the position of P is described by $\vec{r}_P = r\,\hat{u}_r$. Applying Eq. (2.48) on p. 81 of the textbook, we have

$$\vec{v}_P = \dot{r}\,\hat{u}_r + r\vec{\omega}_r \times \hat{u}_r, \tag{1}$$

where $\vec{\omega}_r$ is the angular velocity of the unit vector $\hat{u}_r$. Since the angle θ describes the orientation of $\vec{r}_P$, we have that

$$\vec{\omega}_r = \dot{\theta}\,\hat{k}, \tag{2}$$

where $\hat{k} = \hat{u}_r \times \hat{u}_\theta$. Substituting Eq. (2) into Eq. (1) gives

$$\vec{v}_P = \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta. \tag{3}$$

Finding the magnitude of the vector, we then have that

$$|\vec{v}_P| = \sqrt{\dot{r}^2 + r^2\dot{\theta}^2} \quad\Rightarrow\quad \boxed{|\vec{v}_P| = 65{,}590\ \text{ft/s},} \tag{4}$$

where we recalled that $r = 21{,}000$ ft, $\dot{r} = -22{,}440$ ft/s, and $\dot{\theta} = -2.935$ rad/s. We now observe that

$$\hat{u}_r = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath} \quad\text{and}\quad \hat{u}_\theta = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}, \tag{5}$$

so that Eq. (3) can be rewritten as

$$\vec{v}_P = \underbrace{(\dot{r}\cos\theta - r\dot{\theta}\sin\theta)}_{v_x}\,\hat{\imath} + \underbrace{(\dot{r}\sin\theta + r\dot{\theta}\cos\theta)}_{v_{Py}}\,\hat{\jmath} = (22{,}430\,\hat{\imath} - 61{,}640\,\hat{\jmath})\ \text{ft/s}. \tag{6}$$

Since $\vec{v}_P$ is directed downward and to the right, the orientation of $\vec{v}_P$ is $-\tan^{-1}(|v_{Py}/v_{P_x}|)$:

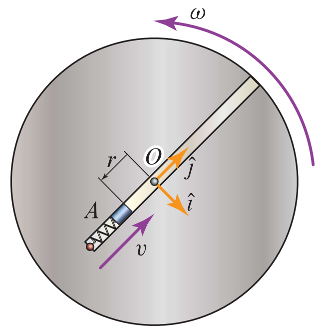
$$\text{Orientation of } \vec{v}_P \text{ from } x \text{ axis} = -\tan^{-1}\left(\left|\frac{\dot{r}\sin\theta + r\dot{\theta}\cos\theta}{\dot{r}\cos\theta - r\dot{\theta}\sin\theta}\right|\right) = -70.01°$$

$$\Rightarrow\quad \boxed{\text{Orientation of } \vec{v} \text{ from } x \text{ axis} = 70.01°\ \text{(cw)},} \tag{7}$$

where, again, $r = 21{,}000$ ft, $\dot{r} = -22{,}440$ ft/s, and $\dot{\theta} = -2.935$ rad/s.

Problem 2.135

A disk rotates about its center, which is the fixed point O. The disk has a straight channel whose centerline passes by O and within which a collar A is allowed to slide. If, when A passes by O, the speed of A relative to the channel is $v = 14\,\mathrm{m/s}$ and is increasing in the direction shown with a rate of $5\,\mathrm{m/s^2}$, determine the acceleration of A given that $\omega = 4\,\mathrm{rad/s}$ and is constant. Express the answer using the component system shown, which rotates with the disk. *Hint:* Apply the equation derived in Prob. 2.122 to the vector describing the position of A relative to O and then let $r = 0$.



Solution

Let $\vec{r}_A$ be the position of A relative to the fixed point O. Using the $(\hat{\imath}, \hat{\jmath})$ component system, $\vec{r}$ can be written as

$$\vec{r}_A = -r\,\hat{\jmath}. \tag{1}$$

Applying the equation derived in Problem 2.122, the acceleration of A is

$$\vec{a}_A = -\ddot{r}\,\hat{\jmath} - 2\vec{\omega}_r \times \dot{r}\,\hat{\jmath} + \dot{\vec{\omega}}_r \times \vec{r}_A + \vec{\omega}_r \times \left(\vec{\omega}_r \times \vec{r}_A\right), \tag{2}$$

where $\vec{\omega}_r$ is the angular velocity of the vector $\vec{r}$. When A is at O, $\vec{r} = \vec{0}$ so that Eq. (2) can be simplified to

$$\vec{a}_A\big|_{r=0} = -\ddot{r}\,\hat{\jmath} - 2\vec{\omega}_r \times \dot{r}\,\hat{\jmath}. \tag{3}$$

Now, we observe that

$$\vec{\omega}_r = \omega\,\hat{k}. \tag{4}$$

Substituting Eq. (4) in Eq. (3), we have

$$\vec{a}_A\big|_{r=0} = -\ddot{r}\,\hat{\jmath} - 2\omega\,\hat{k} \times \dot{r}\,\hat{\jmath} \quad \Rightarrow \quad \vec{a}_A\big|_{r=0} = 2\omega\dot{r}\,\hat{\imath} - \ddot{r}\,\hat{\jmath}. \tag{5}$$

Recalling that $\dot{r} = -v = -14\,\mathrm{m/s}$, $\ddot{r} = -5\,\mathrm{m/s^2}$, and $\omega = 4\,\mathrm{rad/s}$, we can evaluate the last of Eqs. (5) to obtain

$$\boxed{\vec{a}_A\big|_{r=0} = (-112.0\,\hat{\imath} + 5.000\,\hat{\jmath})\,\mathrm{m/s^2}.}$$