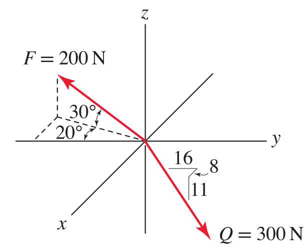
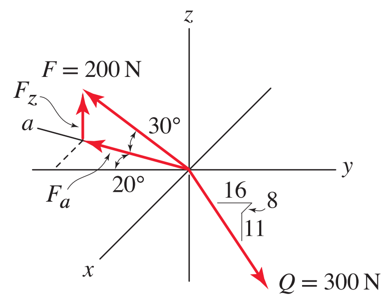
Problem 2.72

Write an expression for each force using Cartesian representation, and evaluate the resultant force vector. Sketch the resultant force in the xyz coordinate system.



Solution

Using the sketch shown, the projections of $\vec{F}$ in the z and a directions are



$$F_z = 200\,\text{N}\sin 30° = 100\,\text{N}, \tag{1}$$
$$F_a = 200\,\text{N}\cos 30° = 173.2\,\text{N}. \tag{2}$$

The component F_a may then be resolved into x and y components as follows

$$F_x = -F_a \sin 20° = -(173.2\,\text{N})\sin 20° = -59.24\,\text{N}, \tag{3}$$
$$F_y = -F_a \cos 20° = -(173.2\,\text{N})\cos 20° = -162.8\,\text{N}, \tag{4}$$

so we may write

$$\vec{F} = (-59.24\,\hat{\imath} - 162.8\,\hat{\jmath} + 100\,\hat{k})\,\text{N}. \tag{5}$$

Using the orientation geometry for $\vec{Q}$ provided in the problem statement, we may write

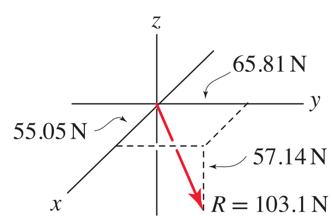
$$\vec{Q} = 300\,\text{N}\left(\frac{8\,\hat{\imath} + 16\,\hat{\jmath} - 11\,\hat{k}}{21}\right) \tag{6}$$
$$= (114.3\,\hat{\imath} + 228.6\,\hat{\jmath} - 157.1\,\hat{k})\,\text{N}. \tag{7}$$

The resultant force vector $\vec{R}$ is

$$\begin{aligned}\vec{R} &= \vec{F} + \vec{Q}\\ &= (-59.24 + 114.3)\,\hat{\imath}\,\text{N}\\ &\quad + (-162.8 + 228.6)\,\hat{\jmath}\,\text{N}\\ &\quad + (100 - 157.1)\,\hat{k}\,\text{N} \quad (8)\\ &= (55.05\,\hat{\imath} + 65.81\,\hat{\jmath} - 57.14\,\hat{k})\,\text{N}. \quad (9)\end{aligned}$$

The magnitude of R is

$$R = \sqrt{(55.05\,\text{N})^2 + (65.81\,\text{N})^2 + (-57.14\,\text{N})^2} \tag{10}$$
$$= 103.1\,\text{N}. \tag{11}$$

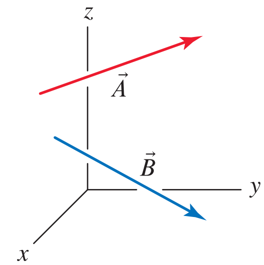


Problem 2.103

(a) Determine the angle between vectors $\vec{A}$ and $\vec{B}$.

(b) Determine the components of $\vec{A}$ parallel and perpendicular to $\vec{B}$.

(c) Determine the vector components of $\vec{A}$ parallel and perpendicular to $\vec{B}$.



$\vec{A} = (-\hat{\imath} + 8\hat{\jmath} + 4\hat{k})\,\mathrm{N}$
$\vec{B} = (\hat{\imath} + 18\hat{\jmath} - 6\hat{k})\,\mathrm{mm}$

Solution

Part (a) Begin by computing the magnitudes of the vectors $\vec{A}$ and $\vec{B}$

$$A = \sqrt{(-1)^2 + (8)^2 + (4)^2}\,\mathrm{N} = 9\,\mathrm{N}, \quad B = \sqrt{(1)^2 + (18)^2 + (-6)^2}\,\mathrm{mm} = 19\,\mathrm{mm}. \tag{1}$$

The angle θ between these vectors is given by

$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right) = \cos^{-1}\left[\frac{(-1\,\mathrm{N})(1\,\mathrm{mm}) + (8\,\mathrm{N})(18\,\mathrm{mm}) + (4\,\mathrm{N})(-6\,\mathrm{mm})}{(9\,\mathrm{N})(19\,\mathrm{mm})}\right] \tag{2}$$

$$= \cos^{-1}\left(\frac{119}{171}\right) = \boxed{45.9^\circ.} \tag{3}$$

Part (b) The parallel component of $\vec{A}$ is given by $A_\parallel$ and the perpendicular component is given by $A_\perp$, such that

$$A_\parallel = \vec{A}\cdot\frac{\vec{B}}{B} = \frac{(-1\,\mathrm{N})(1\,\mathrm{mm}) + (8\,\mathrm{N})(18\,\mathrm{mm}) + (4\,\mathrm{N})(-6\,\mathrm{mm})}{19\,\mathrm{mm}} = 6.26\,\mathrm{N}, \tag{4}$$

$$A_\perp = \sqrt{A^2 - A_\parallel^2} = \sqrt{(9\,\mathrm{N})^2 - (6.263\,\mathrm{N})^2} = 6.46\,\mathrm{N}. \tag{5}$$

Part (c) The vector component of $\vec{A}$ parallel to $\vec{B}$ is given by

$$\vec{A}_\parallel = A_\parallel \cdot \frac{\vec{B}}{B} = (6.263\,\mathrm{N})\left(\frac{(\hat{\imath} + 18\,\hat{\jmath} - 6\,\hat{k})\,\mathrm{mm}}{19\,\mathrm{mm}}\right) = (0.330\,\hat{\imath} + 5.93\,\hat{\jmath} - 1.98\,\hat{k})\,\mathrm{N}. \tag{6}$$

The perpendicular vector component is then

$$\vec{A}_\perp = \vec{A} - \vec{A}_\parallel = (-\hat{\imath} + 8\,\hat{\jmath} + 4\,\hat{k})\,\mathrm{N} - (0.3296\,\hat{\imath} + 5.934\,\hat{\jmath} - 1.978\,\hat{k})\,\mathrm{N},$$
$$= (-1.33\,\hat{\imath} + 2.07\,\hat{\jmath} + 5.98\,\hat{k})\,\mathrm{N}. \tag{7}$$

As a partial check of the accuracy of our results, we evaluate the magnitude of Eq. (7) to obtain

$$A_\perp = \sqrt{(-1.330\,\mathrm{N})^2 + (2.066\,\mathrm{N})^2 + (5.978\,\mathrm{N})^2} = 6.46\,\mathrm{N}, \tag{8}$$

which agrees with the value found in Eq. (5).

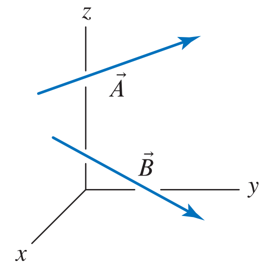
Problem 2.146

(a) Evaluate $\vec{A} \times \vec{B}$.

(b) Evaluate $\vec{B} \times \vec{A}$.

(c) Comment on any differences between the results of Parts (a) and (b).

(d) Use the dot product to show the result of Part (a) is orthogonal to vectors $\vec{A}$ and $\vec{B}$.



$\vec{A} = (-\hat{\imath} + 8\hat{\jmath} + 4\hat{k})$ mm
$\vec{B} = (\hat{\imath} + 18\hat{\jmath} - 6\hat{k})$ mm

Solution

Part (a) Evaluating the cross product $\vec{A} \times \vec{B}$ using the determinant provides

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 8 & 4 \\ 1 & 18 & -6 \end{vmatrix} \text{mm}^2 \tag{1}$$

$$= \left\{ \hat{\imath}\ [(8)(-6) - (4)(18)] - \hat{\jmath}\ [(-1)(-6) - (4)(1)] + \hat{k}\ [(-1)(18) - (8)(1)] \right\} \text{mm}^2 \tag{2}$$

$$= (-120\,\hat{\imath} - 2\,\hat{\jmath} - 26\,\hat{k})\,\text{mm}^2. \tag{3}$$

Part (b) Evaluating the cross product $\vec{B} \times \vec{A}$ using the determinant provides

$$\vec{B} \times \vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 18 & -6 \\ -1 & 8 & 4 \end{vmatrix} \text{mm}^2 \tag{4}$$

$$= \left\{ \hat{\imath}\ [(18)(4) - (-6)(8)] - \hat{\jmath}\ [(1)(4) - (-6)(-1)] + \hat{k}\ [(1)(8) - (18)(-1)] \right\} \text{mm}^2 \tag{5}$$

$$= (120\,\hat{\imath} + 2\,\hat{\jmath} + 26\,\hat{k})\,\text{mm}^2. \tag{6}$$

Part (c) Comparing the results of Parts (a) and (b), observe that $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ are vectors with equal magnitude but opposite direction.

Part (d) Consider the vector $\vec{C} = \vec{A} \times \vec{B}$, where from Part (a), $\vec{C} = (-120\,\hat{\imath} - 2\,\hat{\jmath} - 26\,\hat{k})\,\text{mm}^2$. Noting that two vectors are orthogonal if the dot product between them is zero, we evaluate the following dot products

$$\vec{C} \cdot \vec{A} = (-120\,\text{mm}^2)(-1\,\text{mm}) + (-2\,\text{mm}^2)(8\,\text{mm}) + (-26\,\text{mm}^2)(4\,\text{mm}) = 0, \tag{7}$$

$$\vec{C} \cdot \vec{B} = (-120\,\text{mm}^2)(1\,\text{mm}) + (-2\,\text{mm}^2)(18\,\text{mm}) + (-26\,\text{mm}^2)(-6\,\text{mm}) = 0. \tag{8}$$

Thus,

$$\vec{C} \text{ is orthogonal to both } \vec{A} \text{ and } \vec{B}. \tag{9}$$