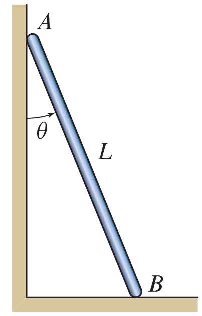
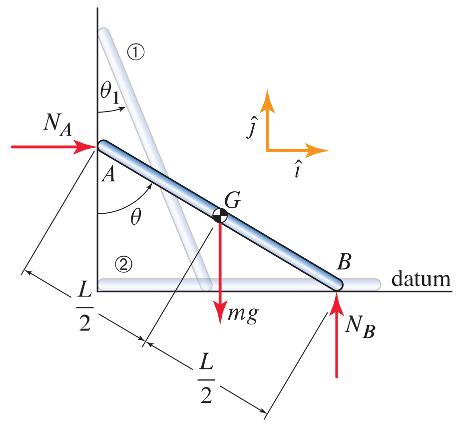
## Problem 8.18

A uniform thin bar $AB$ of length $L = 4$ ft is released from rest at an angle $\theta = \theta_1$. As the bar slides, the ends $A$ and $B$ maintain contact with the surfaces on which they slide. Neglecting friction and knowing that the end $A$ has a speed of 18 ft/s right before hitting the floor, determine $\theta_1$.




### *Solution*

We model the bar as a thin rigid body. We assume that the bar is subject only to its own weight $mg$, which is a conservative force, and to the normal reactions at $A$ and $B$ perpendicular to the wall and the floor, respectively. We denote by $G$ the center of mass of the bar. We denote by ① the position at which the bar is released from rest. We denote by ② the position of the bar right before end $A$ hits the floor. We set the datum for gravity at the floor. We use subscripts 1 and 2 to denote quantities at ① and ② and we observe that the weight of the bar is the only force doing work.




**Balance Principles.** Applying the work-energy principle as a statement of conservation of energy, we have

$$T_1 + V_1 = T_2 + V_2, \tag{1}$$

where $V$ is the potential energy of the bar, and where, denoting by $v_G$ the speed of the center of mass of the bar, by $I_G$ the mass moment of inertia of the bar $G$, and by $\omega_b$ the angular speed of the bar, we have

$$T_1 = \tfrac{1}{2}mv_{G1}^2 + \tfrac{1}{2}I_G\omega_{b1}^2 \quad \text{and} \quad T_2 = \tfrac{1}{2}mv_{G2}^2 + \tfrac{1}{2}I_G\omega_{b2}^2. \tag{2}$$

For $I_G$ we have

$$I_G = \tfrac{1}{12}mL^2. \tag{3}$$

**Force Laws.** Due to the choice of datum,

$$V_1 = \tfrac{1}{2}mgL\cos\theta_1 \quad \text{and} \quad V_2 = 0. \tag{4}$$

**Kinematic Equations.** The bar is released from rest. Also, in ② point $B$ is at a distance $L$ from the wall, i.e., it is at a maximum distance from the wall while its motion is constrained to remain in the horizontal direction. This implies that the speed of $B$ in ② is equal to zero so $B$ is the instantaneous center of rotation for the bar. This implies that $v_{A2} = L\omega_{b2}$, where $v_A$ is the speed of $A$. Therefore,

$$v_{G1} = 0, \quad \omega_{b1} = 0, \quad v_{G2} = \tfrac{1}{2}v_{A2}, \quad \omega_{b2} = v_{A2}/L. \tag{5}$$

**Computation.** Substituting Eqs. (2)–(5) into Eq. (1), we have

$$\tfrac{1}{2}mgL\cos\theta_1 = \tfrac{1}{8}mv_{A2}^2 + \tfrac{1}{24}mv_{A2}^2 \quad \Rightarrow \quad \theta_1 = \cos^{-1}[v_{A2}^2/(3gL)] \quad \Rightarrow \quad \boxed{\theta_1 = 33.02^\circ,} \tag{6}$$

where we have used the data $v_{A2} = 18$ ft/s, $g = 32.2$ ft/s$^2$, and $L = 4$ ft.

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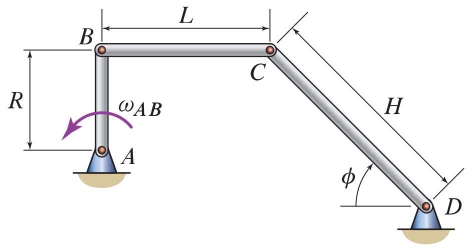
## Problem 8.40

The weights of the uniform thin pin-connected bars $AB$, $BC$, and $CD$ are $W_{AB} = 4\,\text{lb}$, $W_{BC} = 6.5\,\text{lb}$, and $W_{CD} = 10\,\text{lb}$, respectively. Letting $\phi = 47°$, $R = 2\,\text{ft}$, $L = 3.5\,\text{ft}$, and $H = 4.5\,\text{ft}$, and knowing that bar $AB$ rotates at an angular velocity $\omega_{AB} = 4\,\text{rad/s}$, compute the kinetic energy $T$ of the system at the instant shown.




### Solution

The kinetic energy of the system is the sum of the kinetic energies of each individual component, i.e.,

$$T = T_{AB} + T_{BC} + T_{CD}. \tag{1}$$

Bars $AB$ and $CD$ are in fixed axis rotations about $A$ and $D$ respectively. Hence, letting $G$ denote the mass center of bar $BC$, we have

$$T_{AB} = \tfrac{1}{2} I_A \omega_{AB}^2, \quad T_{BC} = \tfrac{1}{2} m_{BC} v_G^2 + \tfrac{1}{2} I_G \omega_{BC}^2, \quad \text{and} \quad T_{CD} = \tfrac{1}{2} I_D \omega_{CD}^2, \tag{2}$$

where $I_A$ is the mass moment of inertia of bar $AB$ about $A$, $I_G$ is the mass moment of inertia of bar $BC$ about its own mass center $G$, and $I_D$ is the mass moment of inertia of bar $CD$ about $D$, i.e.,

$$I_A = \tfrac{1}{12} m_{AB} R^2 + m_{AB}(R/2)^2 = 0.1656\,\text{slug}\cdot\text{ft}^2, \tag{3}$$

$$I_G = \tfrac{1}{12} m_{BC} L^2 = 0.2061\,\text{slug}\cdot\text{ft}^2, \tag{4}$$

$$I_D = \tfrac{1}{12} m_{CD} H^2 + m_{CD}(H/2)^2 = 2.096\,\text{slug}\cdot\text{ft}^2, \tag{5}$$

where, recalling that the acceleration due to gravity is $g = 32.2\,\text{ft/s}^2$, we have used the following numerical data: $m_{AB} = 4\,\text{lb}/g$, $R = 2\,\text{ft}$, $m_{BC} = 6.5\,\text{lb}/g$, $L = 3.5\,\text{ft}$, $m_{CD} = 10\,\text{lb}/g$, and $H = 4.5\,\text{ft}$.

We now determine $\omega_{BC}$, $v_G$, and $\omega_{CD}$. The position of vectors of $B$ relative to $A$, $C$ relative to $B$, and $D$ relative to $C$ are, respectively,

$$\vec{r}_{B/A} = R\,\hat{\jmath}, \quad \vec{r}_{C/B} = L\,\hat{\imath}, \quad \text{and} \quad \vec{r}_{D/C} = H(\cos\phi\,\hat{\imath} - \sin\phi\,\hat{\jmath}). \tag{6}$$

Then we have

$$\vec{v}_B = \omega_{AB}\,\hat{k} \times \vec{r}_{B/A} = -R\omega_{AB}\,\hat{\imath}, \tag{7}$$

$$\vec{v}_C = \vec{v}_B + \omega_{BC}\,\hat{k} \times \vec{r}_{C/B} = -R\omega_{AB}\,\hat{\imath} + \omega_{BC} L\,\hat{\jmath}, \tag{8}$$

$$\vec{v}_D = \vec{v}_C + \omega_{CD}\,\hat{k} \times \vec{r}_{D/C} = (H\omega_{CD}\sin\phi - R\omega_{AB})\,\hat{\imath} + (L\omega_{BC} + H\omega_{CD}\cos\phi)\,\hat{\jmath}. \tag{9}$$

Since $D$ is a fixed point, we must have $\vec{v}_D = \vec{0}$, which then implies

$$\omega_{BC} = -\omega_{AB}\frac{R\cos\phi}{L\sin\phi} = -2.131\,\text{rad/s} \quad \text{and} \quad \omega_{CD} = \omega_{AB}\frac{R}{H\sin\phi} = 2.431\,\text{rad/s}, \tag{10}$$

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where we have used the following numerical data: $\omega_{AB} = 4\,\text{rad/s}$, $R = 2\,\text{ft}$, $L = 3.5\,\text{ft}$, and $\phi = 47°$. Then, using the first of Eqs. (10), we have that the velocity of $G$ is

$$\vec{v}_G = \vec{v}_B + \omega_{BC}\,\hat{k} \times \tfrac{1}{2}\vec{r}_{C/B} = -R\omega_{AB}\,\hat{\imath} - \tfrac{1}{2}R\omega_{AB}\frac{\cos\phi}{\sin\phi}\,\hat{\jmath}$$

$$\Rightarrow \quad v_G^2 = R^2\omega_{AB}^2\left(1 + \tfrac{1}{4}\frac{\cos^2\phi}{\sin^2\phi}\right) = 77.91\,\text{ft}^2/\text{s}^2, \quad (11)$$

where we have used some of the same numerical data listed earlier. Substituting Eq. (3) into the first of Eq. (2) and recalling that $\omega_{AB} = 4\,\text{rad/s}$, we have

$$T_{AB} = 1.325\,\text{ft}\cdot\text{lb}. \quad (12)$$

Substituting the numerical results in Eq. (4), the first of Eqs. (10), and Eq. (11) into the second of Eq. (2), we have

$$T_{BC} = 8.332\,\text{ft}\cdot\text{lb}. \quad (13)$$

Finally, substituting the numerical results in Eq. (5) and in the second of Eqs. (10), we have

$$T_{CD} = 6.193\,\text{ft}\cdot\text{lb}. \quad (14)$$

Keeping in mind that, although results are reported to four significant figures, the full precision of the results obtained thus far is retained in the final result, summing the last three results, we have

$$\boxed{T = 15.85\,\text{ft}\cdot\text{lb}.}$$

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