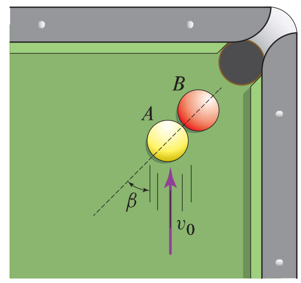
Problem 5.95

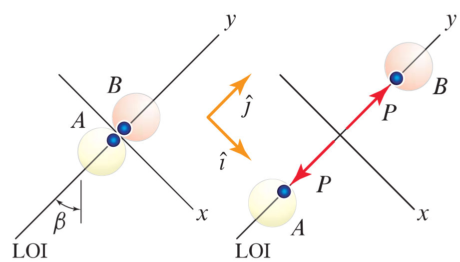
Ball B is stationary when it is hit by an identical ball A as shown, with $\beta = 45°$. The preimpact speed of ball A is $v_0 = 1\,\text{m/s}$.

Determine the postimpact velocity of ball A if the COR of the collision $e = 0.8$.



Solution

We model the impact of A and B as an unconstrained oblique central impact of two particles. The impact-relevant FBD of A and B as a system and of A and B individually is shown at the right, where we have denoted by x an axis perpendicular to the LOI and by y the LOI itself. We will denote the masses of A and B by m_A and m_B, respectively. In addition, following the convention introduced in the textbook, we will use the superscripts $-$ and $+$ to denote quantities computed right before and right after impact.



Balance Principles. As with any unconstrained oblique central impact, we have conservation of momentum for the entire system along the LOI along with conservation of momentum for A and B individually in the direction perpendicular to the LOI:

$$m_A v_{Ay}^- + m_B v_{By}^- = m_A v_{Ay}^+ + m_B v_{By}^+, \tag{1}$$
$$m_A v_{Ax}^- = m_A v_{Ax}^+, \tag{2}$$
$$m_B v_{Bx}^- = m_B v_{Bx}^+, \tag{3}$$

where v_{Ax} and v_{Ay} are the x and y components of the velocity of A, and v_{Bx} and v_{By} are the x and y components of the velocity of B.

Force Laws. The effect of the contact force P between A and B is expressed via the COR equation (along the LOI):

$$v_{Ay}^+ - v_{By}^+ = e(v_{By}^- - v_{Ay}^-). \tag{4}$$

Kinematic Equations. Before impact, A is traveling with a speed v_0 at an angle β with respect to the LOI whereas B is stationary. Hence, we have

$$v_{Ax}^- = -v_0 \sin\beta, \quad v_{Ay}^- = v_0 \cos\beta, \quad v_{Bx}^- = 0, \quad v_{By}^- = 0. \tag{5}$$

Computation. Substituting Eqs. (5) into Eqs. (1)–(4) we obtain

$$m_A v_0 \cos\beta = m_A v_{Ay}^+ + m_B v_{By}^+, \tag{6}$$
$$-v_0 \sin\beta = v_{Ax}^+, \tag{7}$$
$$0 = v_{Bx}^+, \tag{8}$$
$$v_{Ay}^+ - v_{By}^+ = -e v_0 \cos\beta, \tag{9}$$

which is a system of four equations in the four unknowns v_{Ax}^+, v_{Ay}^+, v_{Bx}^+, and v_{By}^+ whose solution is

$$v_{Ax}^+ = -v_0 \sin\beta, \quad v_{Ay}^+ = v_0 \frac{m_A - e m_B}{m_A + m_B} \cos\beta, \quad v_{Bx}^+ = 0, \quad v_{By}^+ = v_0 \frac{m_A(1+e)}{m_A + m_B} \cos\beta. \tag{10}$$

Recalling that $m_A = m_B$, the solution can be simplified to

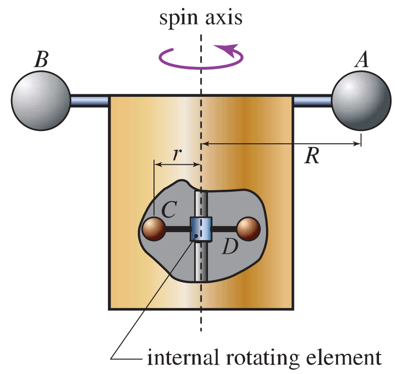
$$v_{Ax}^+ = -v_0 \sin\beta, \quad v_{Ay}^+ = \tfrac{1}{2} v_0 (1-e) \cos\beta, \quad v_{Bx}^+ = 0, \quad v_{By}^+ = \tfrac{1}{2} v_0 (1+e) \cos\beta. \tag{11}$$

Recalling that $v_0 = 1\,\text{m/s}$, $e = 0.8$, and $\beta = 45^\circ$, we can evaluate the postimpact velocity of A to obtain

$$\vec{v}_A^+ = (-0.7071\,\hat{\imath} + 0.07071\,\hat{\jmath})\,\text{m/s}.$$  @ -45°

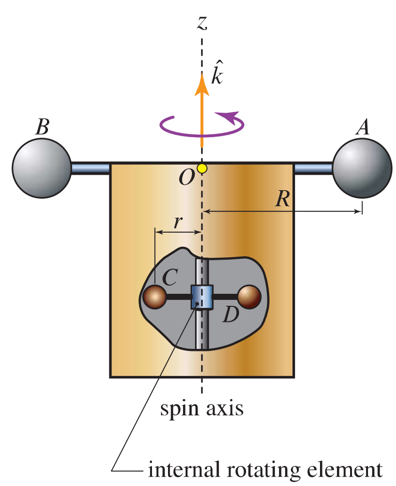
Problem 5.133

The body of the satellite shown has a weight that is negligible with respect to the two spheres A and B that are rigidly attached to it, which weigh 150 lb each. The distance between A and B from the spin axis of the satellite is $R = 3.5$ ft. Inside the satellite there are two spheres C and D weighing 4 lb mounted on a motor that allows them to spin about the axis of the cylinder at a distance $r = 0.75$ ft from the spin axis. Suppose that the satellite is released from rest and that the internal motor is made to spin up the internal masses at an absolute constant time rate of $5.0\,\text{rad/s}^2$ (measured relative to an inertial observer) for a total of 10 s. Treating the system as isolated, determine the angular speed of the satellite at the end of spin-up.



Solution

Referring to the figure at the right, let $\hat{k}$ denote the positive direction of the spin axis and let O be a reference point on the spin axis. We model A, B, C, and D as a system of particles. We observe that this system is isolated. Therefore, the angular momentum of the system is conserved. We define t_1 and t_2 to be the time instants when the C and D are put in motion, and after they have been spun for a total of 10 s, respectively. We use the subscripts 1 and 2 to denote quantities at t_1 and t_2, respectively. We assume that throughout the motion the orientation of the spin axis does not change and that z axis is stationary relative to some inertia reference frame.



Balance Principles. Applying the impulse-momentum principle in the form of conservation of angular momentum, and focusing on the component of the angular momentum along the z axis, which is the spin axis, we have

$$(h_{Oz})_1 = (h_{Oz})_2, \tag{1}$$

where, assuming that the system only rotates about the spins axis,

$$h_{Oz} = (m_A + m_B)\omega_s R^2 + (m_C + m_D)\omega_i r^2, \tag{2}$$

where $\vec{\omega}_s = \omega_s\,\hat{k}$ denotes the angular velocity of the external masses moving with the body of the satellite (the subscript s stands for 'satellite'), and where $\vec{\omega}_i = \omega_i\,\hat{k}$ denotes the angular velocity of the internal masses (the subscript i stands for 'internal').

Force Laws. All forces are accounted for on the FBD (the system is isolated so there are no external forces acting on the system).

Kinematic Equations. Assuming motion of the satellite only about the z axis, the angular acceleration of the internal masses is

$$\vec{\alpha}_i = \alpha_i\,\hat{k}, \tag{3}$$

where $\alpha_i = 5.00\,\text{rad/s}^2$. Assuming that the spin axis does not change orientation, since the system is initially at rest, letting $\tau = t_2 - t_1 = 10$ s, the initial and final angular velocities of the internal masses are

$$\omega_{s1} = 0, \quad \omega_{i1} = 0, \quad \omega_{i2} = \alpha_i\tau. \tag{4}$$

Computation. Substituting Eqs. (2) and Eqs. (4) into Eq. (1), we have

$$0 = (m_A + m_B)R^2\omega_{s2} + (m_C + m_D)r^2\alpha_i\tau \quad \Rightarrow \quad \omega_{s2} = -\frac{(m_C + m_D)r^2}{(m_A + m_B)R^2}\alpha_i\tau. \tag{5}$$

Recalling that $m_C = m_D = 4\,\mathrm{lb}/g$ $g = 32.2\,\mathrm{ft/s^2}$, $r = 0.75\,\mathrm{ft}$, $m_A = m_B = 150\,\mathrm{lb}/g$, $R = 3.5\,\mathrm{ft}$, $\alpha_i = 5.0\,\mathrm{rad/s^2}$, and $\tau = 10\,\mathrm{s}$, we can evaluate the angular speed $|\vec{\omega}_{s2}| = |\omega_{s2}|$ to obtain

$$\boxed{\left|\vec{\omega}_{s2}\right| = 0.06122\,\mathrm{rad/s}.}$$