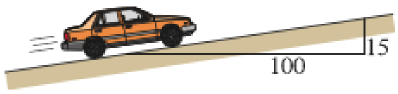
Problem 4.10

Consider a 1500 kg car whose speed is increased by 45 km/h over a distance of 50 m while traveling up an incline with a 15% grade.

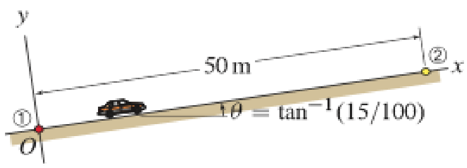
Modeling the car as a particle, determine the work done on the car if the car starts from rest.



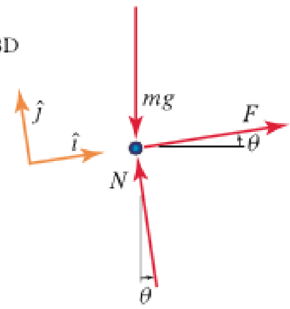
Solution

We model the car as a particle and we define ① and ② to be the positions of the car at the beginning and end of the 50 m stretch, respectively. Referring to the FBD on the right, we assume that the car is subject to its own weight mg, the normal reaction N with the ground, and a propelling force F. Subscripts 1 and 2 will denote quantities at ① and ②, respectively.

coordinate system



FBD



We begin by writing our energy balance.

$$T_1 + U_1 + W = T_2 + U_2$$

T and U are the kinetic and gravitational potential energies at the points indicated by the subscripts, and W is the work done on the car between points 1 and 2, adding energy to the system. $T_1 = 0$ because the car starts from rest, and we'll define the height of Point 1 as zero so that $U_1 = 0$. We then have $W = T_2 + U_2$.

$$T_2 = \frac{1}{2} m v_2^2 \text{ and } U_2 = mgh_2$$

$$v_2 = 45\, km/h = 12.5\, m/s \text{ and } h_2 = 50 * \sin(\tan^{-1}(15/100)) \approx 7.417\, m$$

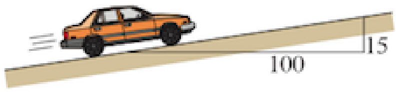
$$W \approx \frac{1}{2}(1500)12.5^2 + 1500(9.81)7.417$$

$$\boxed{W \approx 226.33\, kJ}$$

Problem 4.11

Consider a 1500 kg car whose speed is increased by 45 km/h over a distance of 50 m while traveling up an incline with a 15% grade.

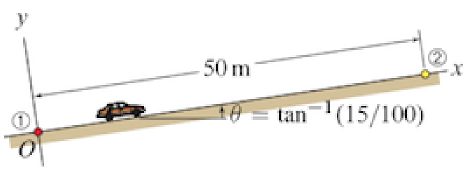
Modeling the car as a particle, determine the work done on the car if the car has an initial speed of 60 km/h.



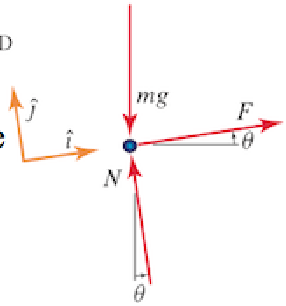
Solution

We model the car as a particle and we define ① and ② to be the positions of the car at the beginning and end of the 50 m stretch, respectively. Referring to the FBD on the right, we assume that the car is subject to its own weight mg, the normal reaction N with the ground, and a propelling force F. Subscripts 1 and 2 will denote quantities at ① and ②, respectively.

coordinate system



FBD



We begin by writing our energy balance.

$$T_1 + U_1 + W = T_2 + U_2$$

T and U are the kinetic and gravitational potential energies at the points indicated by the subscripts, and W is the work done on the car between points 1 and 2, adding energy to the system. We'll define the height of Point 1 as zero so that $U_1 = 0$. We then have $T_1 + W = T_2 + U_2$ -> $W = T_2 + U_2 - T_1$.

$$T_1 = \frac{1}{2} m v_1^2,\ T_2 = \frac{1}{2} m v_2^2, \text{ and } U_2 = mgh_2$$

$$v_1 = 60\ km/h \approx 16.67\ m/s,\ v_2 = 105\ km/h \approx 29.167\ m/s, \text{ and}$$

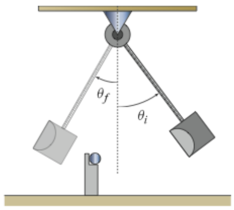
$$h_2 = 50 * \sin(\tan^{-1}(15/100)) \approx 7.417\ m$$

$$W \approx \frac{1}{2}(1500)29.167^2 + 1500(9.81)7.417 - \frac{1}{2}(1500)16.67^2$$

$$\boxed{W \approx 538.76\ kJ}$$

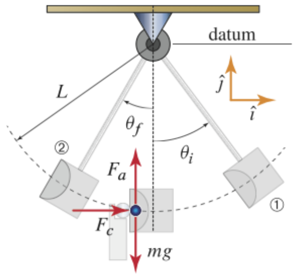
Problem 4.59

The resistance of a material to fracture is assessed with a fracture test. One such test is the *Charpy impact test*, in which the fracture toughness is assessed by measuring the energy required to break a specimen of a specified geometry. This is done by releasing a heavy pendulum from rest at an angle θ_i and by measuring the maximum swing angle θ_f reached by the pendulum after the specimen is broken. Suppose that in an experiment $\theta_i = 45°$, $\theta_f = 23°$, the weight of the pendulum's bob is 3 lb, and the length of the pendulum is 3 ft. Neglecting the mass of any other component of the testing apparatus, assuming that the pendulum's pivot is frictionless, and treating the pendulum's bob as a particle, determine the fracture energy of the specimen tested. Assume that the fracture energy is the energy required to break the specimen.



Solution

We model the bob as a particle subject to its own weight mg, the tension in the pendulum arm F_a, and the contact force with the specimen F_c. Clearly, this force is considered equal to zero when the bob is not in contact with the specimen. We denote by ① the position at which the bob is released. We denote by ② the position at which the bob stops. We use subscripts 1 and 2 to denote quantities at ① and ②, respectively. We observe that F_a does no work because the arm can be modeled as inextensible. Therefore, all of the work done between ① and ② is due to the weight mg, which is conservative, and the contact force with the specimen. In absolute value, the latter work corresponds to the energy required to break the specimen.



Balance Principles. Applying the work-energy principle between ① and ②, we have

$$T_1 + V_1 + (U_{1\text{-}2})_{\text{nc}} = T_2 + V_2, \tag{1}$$

where V is the potential energy of the bob, $(U_{1\text{-}2})_{\text{nc}}$ is the energy required to break the specimen, and where, denoting by v the speed of the bob,

$$T_1 = \tfrac{1}{2}mv_1^2 \quad \text{and} \quad T_2 = \tfrac{1}{2}mv_2^2. \tag{2}$$

Force Laws. We do not provide an expression for $(U_{1\text{-}2})_{\text{nc}}$ since this is the quantity we want to determine. As for V, due to the choice of datum and denoting by L the length of the arm of the pendulum, we have

$$V_1 = -mgL\cos\theta_i \quad \text{and} \quad V_2 = -mgL\cos\theta_f. \tag{3}$$

Kinematic Equations. The bob is released from rest in ① and comes to a stop in ②. So

$$v_1 = 0 \quad \text{and} \quad v_2 = 0. \tag{4}$$

Computation. Substituting Eqs. (2)–(4) into Eq. (1), we have

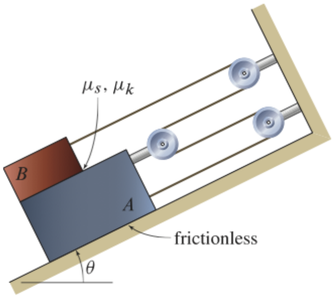
$$-mgL\cos\theta_i + (U_{1\text{-}2})_{\text{nc}} = -mgL\cos\theta_f \quad \Rightarrow \quad (U_{1\text{-}2})_{\text{nc}} = mgL(\cos\theta_i - \cos\theta_f). \tag{5}$$

The quantity $(U_{1\text{-}2})_{\text{nc}}$ is the work done on the pendulum bob by the specimen. Hence the energy required to break the specimen is the negative of $(U_{1\text{-}2})_{\text{nc}}$. Keeping this in mind, and recalling that $m = 3\,\text{lb}/g$, $g = 32.2\,\text{ft/s}^2$, $L = 3\,\text{ft}$, $\theta_i = 45°$, and $\theta_f = 23°$, we can evaluate $(U_{1\text{-}2})_{\text{nc}}$ to obtain

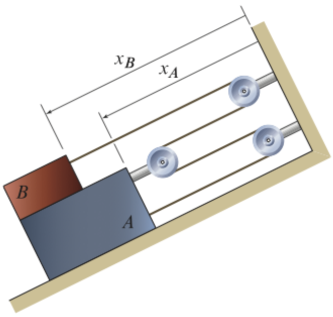
$$\boxed{\text{Energy required to break the specimen} = 1.921\,\text{ft}\cdot\text{lb}.}$$

Problem 4.84

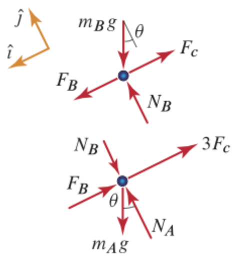
Solve Example 4.14 by applying the work-energy principle to each block individually, and show that the net work done by the cord on the two blocks is zero.



Solution



We denote by ① the release position of the system. We denote by ② the position of the system after the blocks have moved a relative distance d. We consider A and B as a system of two particles and we sketch an FBD for each of these particles between ① and ②. The forces $m_A g$ and $m_B g$ are the weights of A and B, respectively. N_A is the normal reaction between A and the incline. N_B is the normal reaction between A and B. F_c is the tension in the cord. F_B is the friction force between A and B. We will apply the work-energy principle to A and B individually. We will use subscript 1 and 2 to denote quantities in ① and ②, respectively.



Balance Principles. Applying the work-energy principle to A and B, we have

$$T_{A1} + V_{A1} + [(U_{1\text{-}2})_{\text{nc}}]_A = T_{A2} + V_{A2}, \tag{1}$$
$$T_{B1} + V_{B1} + [(U_{1\text{-}2})_{\text{nc}}]_B = T_{B2} + V_{B2}, \tag{2}$$

where V_A and V_B are the potential energies of A and B, respectively, and where, denoting by v_A and v_B the speeds of A and B, respectively,

$$T_{A1} = \tfrac{1}{2} m_A v_{A1}^2, \quad T_{B1} = \tfrac{1}{2} m_B v_{B1}^2, \qquad T_{A2} = \tfrac{1}{2} m_A v_{A2}^2, \quad T_{B2} = \tfrac{1}{2} m_B v_{B2}^2. \tag{3}$$

In addition, observing that F_c and F_B are the only nonconservative forces that do work because the motion of A and B is only along the x direction, we have

$$[(U_{1\text{-}2})_{\text{nc}}]_A = \int_{x_{A1}}^{x_{A2}} -(F_B + 3F_c)\,dx_A \quad \text{and} \quad [(U_{1\text{-}2})_{\text{nc}}]_B = \int_{x_{B1}}^{x_{B2}} (F_B - F_c)\,dx_B. \tag{4}$$

Force Laws. Choosing the datum for gravity at ①, we have

$$V_{A1} = 0, \quad V_{B1} = 0, \quad V_{A2} = -m_A g (x_{A2} - x_{A1}) \sin\theta, \quad V_{B2} = -m_B g (x_{B2} - x_{B1}) \sin\theta. \tag{5}$$

To determine the work of the internal force, we observe that $F_B = \mu_k N_B$. To determine N_B, summing the forces acting on B in the y direction, we observe that $N_B - m_B g \cos\theta = 0$ because B does not move in the y direction. Therefore, we have

$$F_B = \mu_k m_B g \cos\theta. \tag{6}$$

Kinematic Equations. The system is released from rest, so

$$v_{A1} = 0 \quad \text{and} \quad v_{B1} = 0. \tag{7}$$

Denoting by L the length of the cord,

$$L = 3x_A + x_B = \text{constant} \quad \Rightarrow \quad 3x_{A1} + x_{B1} = 3x_{A2} + x_{B2} \quad \Rightarrow \quad x_{B2} - x_{B1} = -3(x_{A2} - x_{A1}). \tag{8}$$

Now we recall that the distance of A relative to B in ② is d, i.e., $(x_{A2} - x_{A1}) - (x_{B2} - x_{B1}) = d$. Using this fact and the last of Eqs. (8), we conclude that

$$x_{A2} - x_{A1} = \tfrac{1}{4}d \quad \text{and} \quad x_{B2} - x_{B1} = -\tfrac{3}{4}d. \tag{9}$$

In addition, taking the differential and the time derivative of the first of Eqs. (8), we have

$$dx_B = -3\,dx_A \quad \text{and} \quad v_B = 3v_A, \tag{10}$$

where, in the last of Eqs. (10), we have accounted for the fact that the speed of an object is nonnegative.

Computation. Using Eqs. (7) and the last of Eqs. (10), we can rewrite Eq. (3) as follows:

$$T_{A1} = 0, \quad T_{B1} = 0, \quad T_{A2} = \tfrac{1}{2}m_A v_{A2}^2, \quad T_{B2} = \tfrac{9}{2}m_B v_{A2}^2. \tag{11}$$

Using the last of Eqs. (9), we can rewrite Eqs. (5) as follows:

$$V_{A1} = 0, \quad V_{B1} = 0, \quad V_{A2} = -\tfrac{1}{4}m_A g d \sin\theta, \quad V_{B2} = \tfrac{3}{4}m_B g d \sin\theta. \tag{12}$$

Substituting (where appropriate) Eqs. (4), (11), and (12) into Eqs. (1) and (2), we have

$$\int_{x_{A1}}^{x_{A2}} -F_B\,dx_A - \int_{x_{A1}}^{x_{A2}} 3F_c\,dx_A = \tfrac{1}{2}m_A v_{A2}^2 - \tfrac{1}{4}m_A g d \sin\theta, \tag{13}$$

$$\int_{x_{B1}}^{x_{B2}} F_B\,dx_B - \int_{x_{B1}}^{x_{B2}} F_c\,dx_B = \tfrac{9}{2}m_B v_{A2}^2 + \tfrac{3}{4}m_B g d \sin\theta. \tag{14}$$

We now use the first of Eqs. (10) to perform a change of variable of integration for the integrals on the left-hand side of Eq. (14) and rewrite Eq. (14) as follows:

$$\int_{x_{A1}}^{x_{A2}} -3F_B\,dx_A + \int_{x_{A1}}^{x_{A2}} 3F_c\,dx_A = \tfrac{9}{2}m_B v_{A2}^2 + \tfrac{3}{4}m_B g d \sin\theta. \tag{15}$$

> Except for its sign, the second term on the left-hand side of Eq. (13) (the work done by the tension in the cord on A) is equal to the corresponding term in Eq. (15) (the work done by the tension in the cord on B). Thus the net work done by the tension in the cord on the system is equal to zero.

Summing Eqs. (13) and (15) and using Eqs. (6) and the first of Eqs. (9), we have

$$-\mu_k m_B g d \cos\theta = \tfrac{1}{2}(m_A + 9m_B)v_{A2}^2 - \tfrac{1}{4}(m_A - 3m_B) g d \sin\theta. \tag{16}$$

Solving the above equation for v_{A2}, we have

$$v_{A2} = \sqrt{\frac{2gd}{m_A + 9m_B}\left[\tfrac{1}{4}(m_A - 3m_B)\sin\theta - \mu_k m_B \cos\theta\right]}. \tag{17}$$

Recalling that $m_A = 4$ kg, $m_B = 1$ kg, $\theta = 30°$, $\mu_k = 0.1$, and $d = 0.35$ m, we can evaluate v_{A2} and then v_{B2} (using the last of Eqs. (10)) to obtain $v_{A2} = 0.1424$ m/s and $v_{B2} = 0.4273$ m/s. The problem requires us to determine the velocities of A and B. Observing that A moves in the positive x direction and that B moves in the opposite direction, expressing our answer in vector form, we have

$$\vec{v}_{A2} = 0.1424\,\hat{\imath}\ \text{m/s} \quad \text{and} \quad \vec{v}_{B2} = -0.4273\,\hat{\imath}\ \text{m/s}$$

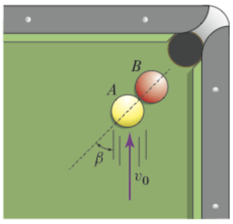
@θ

as expected.

Problem 5.94

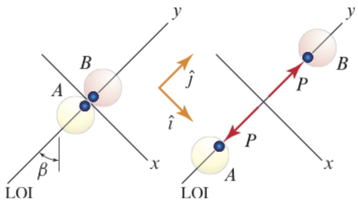
Ball B is stationary when it is hit by an identical ball A as shown, with $\beta = 45°$. The preimpact speed of ball A is $v_0 = 1\,\mathrm{m/s}$.

Determine the postimpact velocity of ball B if the COR of the collision $e = 1$.



Solution

We model the impact of A and B as an unconstrained oblique central impact of two particles. The impact-relevant FBD of A and B as a system and of A and B individually is shown at the right, where we have denoted by x an axis perpendicular to the LOI and by y the LOI itself. We will denote the masses of A and B by m_A and m_B, respectively. In addition, following the convention introduced in the textbook, we will use the superscripts $-$ and $+$ to denote quantities computed right before and right after impact.



Balance Principles. As with any unconstrained oblique central impact, we have conservation of momentum for the entire system along the LOI along with conservation of momentum for A and B individually in the direction perpendicular to the LOI:

$$m_A v_{Ay}^- + m_B v_{By}^- = m_A v_{Ay}^+ + m_B v_{By}^+, \tag{1}$$

$$m_A v_{Ax}^- = m_A v_{Ax}^+, \tag{2}$$

$$m_B v_{Bx}^- = m_B v_{Bx}^+, \tag{3}$$

where v_{Ax} and v_{Ay} are the x and y components of the velocity of A, and v_{Bx} and v_{By} are the x and y components of the velocity of B.

Force Laws. The effect of the contact force P between A and B is expressed via the COR equation (along the LOI):

$$v_{Ay}^+ - v_{By}^+ = e(v_{By}^- - v_{Ay}^-). \tag{4}$$

Kinematic Equations. Before impact, A is traveling with a speed v_0 at an angle β with respect to the LOI whereas B is stationary. Hence, we have

$$v_{Ax}^- = -v_0 \sin\beta, \quad v_{Ay}^- = v_0 \cos\beta, \quad v_{Bx}^- = 0, \quad v_{By}^- = 0. \tag{5}$$

Computation. Substituting Eqs. (5) into Eqs. (1)–(4) we obtain

$$m_A v_0 \cos\beta = m_A v_{Ay}^+ + m_B v_{By}^+, \tag{6}$$

$$-v_0 \sin\beta = v_{Ax}^+, \tag{7}$$

$$0 = v_{Bx}^+, \tag{8}$$

$$v_{Ay}^+ - v_{By}^+ = -e v_0 \cos\beta, \tag{9}$$

which is a system of four equations in the four unknowns v_{Ax}^+, v_{Ay}^+, v_{Bx}^+, and v_{By}^+ whose solution is

$$v_{Ax}^+ = -v_0 \sin\beta, \quad v_{Ay}^+ = v_0 \frac{m_A - e m_B}{m_A + m_B} \cos\beta, \quad v_{Bx}^+ = 0, \quad v_{By}^+ = v_0 \frac{m_A(1+e)}{m_A + m_B} \cos\beta. \tag{10}$$

Recalling that $m_A = m_B$, the solution can be simplified to

$$v_{Ax}^+ = -v_0 \sin\beta, \quad v_{Ay}^+ = \tfrac{1}{2} v_0 (1-e) \cos\beta, \quad v_{Bx}^+ = 0, \quad v_{By}^+ = \tfrac{1}{2} v_0 (1+e) \cos\beta. \tag{11}$$

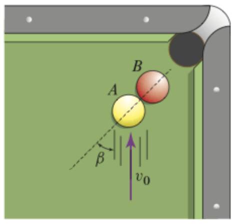
Recalling that $v_0 = 1\,\text{m/s}$, $e = 1$, and $\beta = 45^\circ$, we can evaluate the postimpact velocity of B to obtain

$$\boxed{\vec{v}_B^+ = 0.7071\,\hat{\jmath}\,\text{m/s.}}$$

Problem 5.95

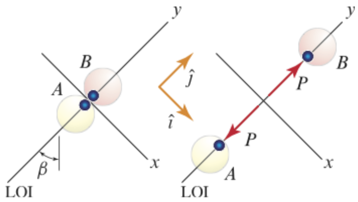
Ball B is stationary when it is hit by an identical ball A as shown, with $\beta = 45°$. The preimpact speed of ball A is $v_0 = 1\,\text{m/s}$.

Determine the postimpact velocity of ball A if the COR of the collision $e = 0.8$.



Solution

We model the impact of A and B as an unconstrained oblique central impact of two particles. The impact-relevant FBD of A and B as a system and of A and B individually is shown at the right, where we have denoted by x an axis perpendicular to the LOI and by y the LOI itself. We will denote the masses of A and B by m_A and m_B, respectively. In addition, following the convention introduced in the textbook, we will use the superscripts $-$ and $+$ to denote quantities computed right before and right after impact.



Balance Principles. As with any unconstrained oblique central impact, we have conservation of momentum for the entire system along the LOI along with conservation of momentum for A and B individually in the direction perpendicular to the LOI:

$$m_A v_{Ay}^- + m_B v_{By}^- = m_A v_{Ay}^+ + m_B v_{By}^+, \tag{1}$$
$$m_A v_{Ax}^- = m_A v_{Ax}^+, \tag{2}$$
$$m_B v_{Bx}^- = m_B v_{Bx}^+, \tag{3}$$

where v_{Ax} and v_{Ay} are the x and y components of the velocity of A, and v_{Bx} and v_{By} are the x and y components of the velocity of B.

Force Laws. The effect of the contact force P between A and B is expressed via the COR equation (along the LOI):

$$v_{Ay}^+ - v_{By}^+ = e(v_{By}^- - v_{Ay}^-). \tag{4}$$

Kinematic Equations. Before impact, A is traveling with a speed v_0 at an angle β with respect to the LOI whereas B is stationary. Hence, we have

$$v_{Ax}^- = -v_0 \sin\beta, \quad v_{Ay}^- = v_0 \cos\beta, \quad v_{Bx}^- = 0, \quad v_{By}^- = 0. \tag{5}$$

Computation. Substituting Eqs. (5) into Eqs. (1)–(4) we obtain

$$m_A v_0 \cos\beta = m_A v_{Ay}^+ + m_B v_{By}^+, \tag{6}$$
$$-v_0 \sin\beta = v_{Ax}^+, \tag{7}$$
$$0 = v_{Bx}^+, \tag{8}$$
$$v_{Ay}^+ - v_{By}^+ = -e v_0 \cos\beta, \tag{9}$$

which is a system of four equations in the four unknowns v^+_{Ax}, v^+_{Ay}, v^+_{Bx}, and v^+_{By} whose solution is

$$v^+_{Ax} = -v_0 \sin\beta, \quad v^+_{Ay} = v_0 \frac{m_A - e m_B}{m_A + m_B} \cos\beta, \quad v^+_{Bx} = 0, \quad v^+_{By} = v_0 \frac{m_A(1+e)}{m_A + m_B} \cos\beta. \tag{10}$$

Recalling that $m_A = m_B$, the solution can be simplified to

$$v^+_{Ax} = -v_0 \sin\beta, \quad v^+_{Ay} = \tfrac{1}{2} v_0 (1-e) \cos\beta, \quad v^+_{Bx} = 0, \quad v^+_{By} = \tfrac{1}{2} v_0 (1+e) \cos\beta. \tag{11}$$

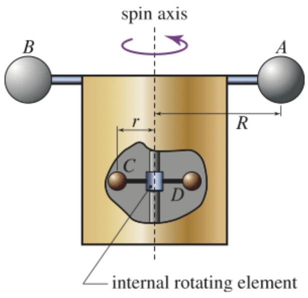
Recalling that $v_0 = 1\,\mathrm{m/s}$, $e = 0.8$, and $\beta = 45^\circ$, we can evaluate the postimpact velocity of A to obtain

$$\boxed{\vec{v}^+_A = (-0.7071\,\hat{\imath} + 0.07071\,\hat{\jmath})\,\mathrm{m/s}.}$$

$@ - 45^\circ$

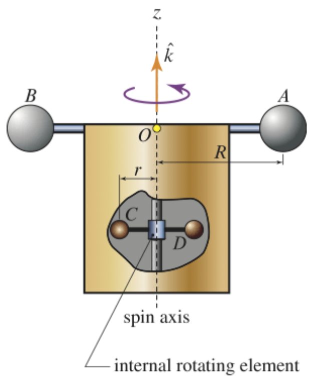
Problem 5.133

The body of the satellite shown has a weight that is negligible with respect to the two spheres A and B that are rigidly attached to it, which weigh 150 lb each. The distance between A and B from the spin axis of the satellite is $R = 3.5$ ft. Inside the satellite there are two spheres C and D weighing 4 lb mounted on a motor that allows them to spin about the axis of the cylinder at a distance $r = 0.75$ ft from the spin axis. Suppose that the satellite is released from rest and that the internal motor is made to spin up the internal masses at an absolute constant time rate of 5.0 rad/s^2 (measured relative to an inertial observer) for a total of 10 s. Treating the system as isolated, determine the angular speed of the satellite at the end of spin-up.



Solution

Referring to the figure at the right, let $\hat{k}$ denote the positive direction of the spin axis and let O be a reference point on the spin axis. We model A, B, C, and D as a system of particles. We observe that this system is isolated. Therefore, the angular momentum of the system is conserved. We define t_1 and t_2 to be the time instants when the C and D are put in motion, and after they have been spun for a total of 10 s, respectively. We use the subscripts 1 and 2 to denote quantities at t_1 and t_2, respectively. We assume that throughout the motion the orientation of the spin axis does not change and that z axis is stationary relative to some inertia reference frame.



Balance Principles. Applying the impulse-momentum principle in the form of conservation of angular momentum, and focusing on the component of the angular momentum along the z axis, which is the spin axis, we have

$$(h_{Oz})_1 = (h_{Oz})_2, \tag{1}$$

where, assuming that the system only rotates about the spins axis,

$$h_{Oz} = (m_A + m_B)\omega_s R^2 + (m_C + m_D)\omega_i r^2, \tag{2}$$

where $\vec{\omega}_s = \omega_s\,\hat{k}$ denotes the angular velocity of the external masses moving with the body of the satellite (the subscript s stands for 'satellite'), and where $\vec{\omega}_i = \omega_i\,\hat{k}$ denotes the angular velocity of the internal masses (the subscript i stands for 'internal').

Force Laws. All forces are accounted for on the FBD (the system is isolated so there are no external forces acting on the system).

Kinematic Equations. Assuming motion of the satellite only about the z axis, the angular acceleration of the internal masses is

$$\vec{\alpha}_i = \alpha_i\,\hat{k}, \tag{3}$$

where $\alpha_i = 5.00\,\text{rad/s}^2$. Assuming that the spin axis does not change orientation, since the system is initially at rest, letting $\tau = t_2 - t_1 = 10$ s, the initial and final angular velocities of the internal masses are

$$\omega_{s1} = 0, \quad \omega_{i1} = 0, \quad \omega_{i2} = \alpha_i\tau. \tag{4}$$

Computation. Substituting Eqs. (2) and Eqs. (4) into Eq. (1), we have

$$0 = (m_A + m_B)R^2\omega_{s2} + (m_C + m_D)r^2\alpha_i\tau \quad \Rightarrow \quad \omega_{s2} = -\frac{(m_C + m_D)r^2}{(m_A + m_B)R^2}\alpha_i\tau. \tag{5}$$

Recalling that $m_C = m_D = 4\,\mathrm{lb}/g$ $g = 32.2\,\mathrm{ft/s^2}$, $r = 0.75\,\mathrm{ft}$, $m_A = m_B = 150\,\mathrm{lb}/g$, $R = 3.5\,\mathrm{ft}$, $\alpha_i = 5.0\,\mathrm{rad/s^2}$, and $\tau = 10\,\mathrm{s}$, we can evaluate the angular speed $|\vec{\omega}_{s2}| = |\omega_{s2}|$ to obtain

$$\boxed{|\vec{\omega}_{s2}| = 0.06122\,\mathrm{rad/s}.}$$