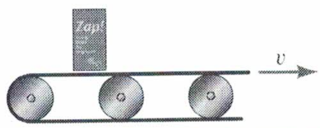
Problem 3.2

An object is lowered very slowly onto a conveyor belt that is moving to the right. What is the direction of the friction force acting on the object at the instant the object touches the belt?



Solution

The friction force will be directed to the right. The horizontal velocity of the object as it is lowered onto the belt is equal to zero. Also, the vertical velocity of the object is negligible. Therefore, the relative velocity of the object with respect to the belt is horizontal and pointing to the left. Since the kinetic friction force opposes relative motion, then this force must point to the right.

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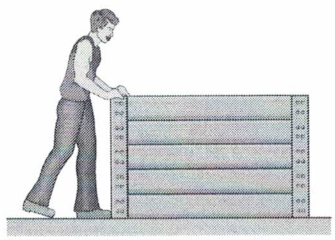
Problem 3.3

A person is trying to move a heavy crate by pushing on it. While the person is pushing, what is the resultant force acting on the crate if the crate does not move?



Solution

The resultant force on the crate is equal to zero. If the object is not moving, then its acceleration is equal to zero and Newton's second law dictates that the total force acting on the object be equal to zero.

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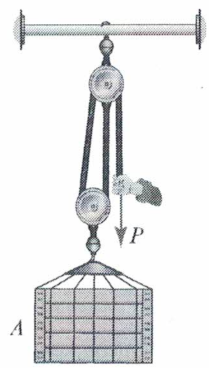
Problem 3.4

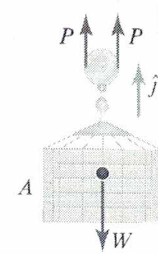
A person is lifting a 75 lb crate A by applying a constant force $P = 40$ lb to the pulley system shown. Neglecting friction and the inertia of the pulleys, determine the acceleration of the crate. Treat all rope segments as purely vertical.



for FBD

Solution

We neglect the motion of the crate in the horizontal direction, we neglect the inertia of the rope and of the pulleys, and we model the rope as inextensible. These assumptions allow us to say that the tension in the rope is uniform along the rope and equal to the force applied by the person. Performing an imaginary cut along a horizontal line placed right above the lower pulley, we have that the FBD of the system, modeled as particle subject to gravity and the force in the rope, is that shown in the figure at the right.



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Balance Principles. Applying Newton's second law in the vertical direction, we have

$$\sum F_y: \quad 2P - W = \frac{W}{g} a_{Ay}, \tag{1}$$

5

where W/g is the mass of the crate and a_{Ay} is the vertical component of the acceleration of the crate.

Force Laws. All forces have been accounted for on the FBD.

Kinematic Equations. We do not need any special kinematic equation since we want to determine a_{Ay} directly.

Computation. Solving Eq. (1) for a_{Ay}, we have

$$a_{Ay} = g\left(\frac{2P}{W} - 1\right). \tag{2}$$

5

Recalling that $g = 32.2\,\text{ft/s}^2$, $P = 40$ lb, and $W = 75$ lb, expressing our answer to three significant figures and in vector form, we can evaluate the result in Eq. (2) to obtain

$$\boxed{\vec{a}_A = 2.147\,\hat{\jmath}\ \text{ft/s}^2.}$$

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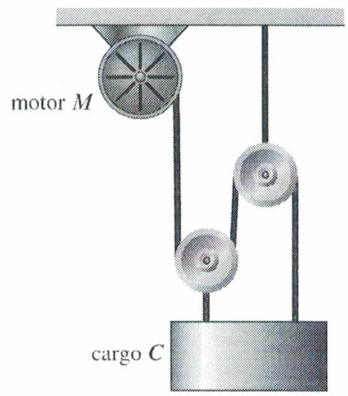
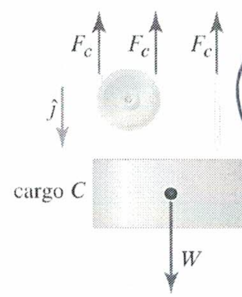
Problem 3.5

The motor M is at rest when someone flips a switch and it starts pulling in the rope. The acceleration of the rope is uniform and is such that it takes 1 s to achieve a retraction rate of 4 ft/s. After 1 s the retraction rate becomes constant. Determine the tension in the rope during and after the initial 1 s interval. The cargo C weighs 130 lb, the weight of the ropes and pulleys is negligible, and friction in the pulleys is negligible.



Solution

We neglect the horizontal motion of the cargo C. We also neglect the inertia of the pulleys so the tension in the rope is uniform. Since we are neglecting the inertia of the pulleys, we analyze the system obtained by cutting along a horizontal line passing between the two pulleys. Modeling this system as a particle subject only to gravity and the tension in the rope, we have the FBD shown at the right.



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Balance Principles. Applying Newton's second law in the vertical direction, we have

$$\sum F_y\colon \quad -3F_c + W = \frac{W}{g} a_{Cy}, \qquad (1)$$

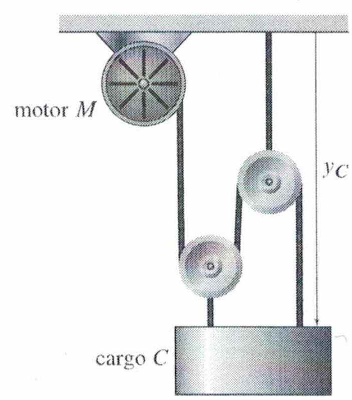
5

where F_c is the tension in the rope, W/g is the mass of C, and a_{Cy} is the vertical component of the acceleration of C.

Force Laws. All forces are accounted for on the FBD.

Kinematic Equations. Using the coordinate system shown at the right, the length of the cord being retracted can be expressed as

$$L = 3y_C + \text{constants}. \qquad (2)$$



Differentiating Eq. (2) twice with respect to time and observing that $a_{Cy} = \ddot{y}_C$, we have

$$\ddot{L} = 3\ddot{y}_C \quad \Rightarrow \quad \ddot{y}_C = \ddot{L}/3 \quad \Rightarrow \quad a_{Cy} = \ddot{L}/3. \qquad (3)$$

5

To determine $\ddot{L}$, we let $t_0 = 1$ s and we recall that the cord starts from rest at $t = 0$, that $\ddot{L}$ is constant, and that, for $t = t_0$, the rate of retraction is $\dot{L}_0 = -4$ ft/s (the minus sign indicates that the cord is getting shorter). Therefore, applying constant acceleration equations, $\ddot{L} = \dot{L}_0/t_0 = -4\,\text{ft/s}^2$. In addition, for $t > t_0$, the rate of retraction becomes constant and $\ddot{L} = 0$. In summary, we have

$$\ddot{L} = \begin{cases} \dfrac{\dot{L}_0}{t_0} = -4\,\text{ft/s}^2 & \text{for } 0 \le t \le t_0, \\ 0 & \text{for } t > t_0. \end{cases} \qquad (4)$$

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Computation. Substituting the last of Eqs. (3) into Eq. (1) and solving for the tension in the cord F_c,

$$F_c = \frac{W}{3}\left(1 - \frac{\ddot{L}}{3g}\right). \tag{5}$$

Recalling that $W = 130\,\text{lb}$, $g = 32.2\,\text{ft/s}^2$, and using Eq. (4), we can evaluate Eq. (5) to obtain

$$\boxed{F_c = \begin{cases} 45.13\,\text{lb} & \text{for } 0 \leq t \leq 1\,\text{s}, \\ 43.33\,\text{lb} & \text{for } t > 1\,\text{s}. \end{cases}}$$

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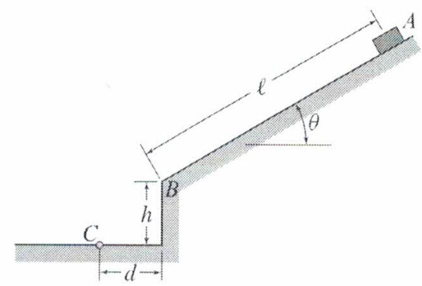
Problem 3.15

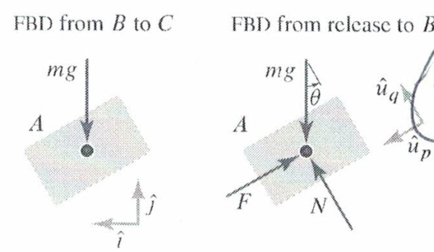
A suitcase is released from rest at A on the $\theta = 30°$ ramp. It slides a distance $\ell = 25$ ft and then goes over the edge at B and drops a height $h = 5$ ft. Determine the horizontal distance d to the landing spot at C.

Assume that the coefficient of static friction is insufficient to prevent slipping and that the coefficient of kinetic friction on the incline between A and B is $\mu_k = 0.3$.



Solution

We model the motion of the suitcase A from B to C as projectile motion. This motion is completely determined by the velocity of A at B. To determine this velocity, we study the sliding motion of A down the incline by modeling A as a particle subject to its own weight mg, the normal reaction N with the incline, and the friction force F (see figure at the right). Between the release point and B we use the component system $\hat{u}_p$ and $\hat{u}_q$ aligned with the incline. Between B and C we use the component system $\hat{\imath}$ and $\hat{\jmath}$, with $\hat{\jmath}$ directed opposite to gravity.



Balance Principles. Applying Newton's second law in the p and q directions, we have

$$\sum F_p: \quad mg\sin\theta - F = ma_{Ap}, \tag{1}$$

$$\sum F_q: \quad N - mg\cos\theta = ma_{Aq}, \tag{2}$$

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where a_{Ap} and a_{Aq} are the p and q components of the the acceleration of the crate, respectively.

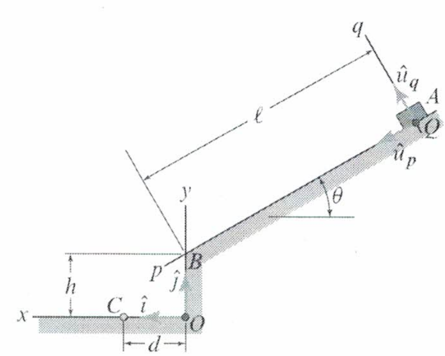
Force Laws. Because A is sliding,

$$F = \mu_k N. \tag{3}$$

Kinematic Equations. Since A does not move perpendicular to the incline,

$$a_{Ap} = \ddot{p} \quad \text{and} \quad a_{Aq} = 0. \tag{4}$$

Also, for future use, we set the origin Q of the pq coordinate system at the release position of A (see figure at the right). Therefore we have that $\dot{p} = 0$ for $p = 0$. In addition, when studying the projectile motion of A, we will be using the xy Cartesian coordinate system with origin at O. Since we will first obtain the velocity of A at B in the pq coordinate system, we will then need to express this result in the xy coordinate system. To do so, we express unit vectors $\hat{u}_p$ and $\hat{u}_q$ in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$:



$$\hat{u}_p = \cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath} \quad \text{and} \quad \hat{u}_q = \sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}. \tag{5}$$

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Computation. Substituting Eqs. (3) and (4) into Eqs. (1) and (2), we have

$$mg\sin\theta - \mu_k N = m\ddot{p} \quad \text{and} \quad N - mg\cos\theta = 0. \tag{6}$$

These equations form a system of two equations in the two unknowns $\ddot{p}$ and N whose solution is

$$\ddot{p} = g(\sin\theta - \mu_k\cos\theta) \quad \text{and} \quad N = mg\cos\theta. \tag{7}$$

To determine the velocity of A at B, we use the chain rule to write $\ddot{p} = \dot{p}\,d\dot{p}/dp$. Using the first of Eqs. (6), and keeping in mind that the motion from Q to B is only in the positive p direction, we can then write

$$\dot{p}\,d\dot{p} = g(\sin\theta - \mu_k\cos\theta)\,dp \quad \Rightarrow \quad \int_0^{v_B}\dot{p}\,d\dot{p} = \int_0^{\ell} g(\sin\theta - \mu_k\cos\theta)\,dp, \tag{8}$$

where v_B is the speed of A at B, and where we accounted for the fact that $\dot{p} = 0$ for $p = 0$. Carrying out the integration and solving for v_B gives

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$$\tfrac{1}{2}v_B^2 = g(\sin\theta - \mu_k\cos\theta)\ell \quad \Rightarrow \quad v_B = \sqrt{2g(\sin\theta - \mu_k\cos\theta)\ell}. \tag{9}$$

We now observe that the velocity of A at B is $\vec{v}_B = v_B\,\hat{u}_p$. Therefore, using the first of Eqs. (5) and the last of Eqs. (9), we have

$$\vec{v}_B = \sqrt{2g(\sin\theta - \mu_k\cos\theta)\ell}\,(\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath}). \tag{10}$$

We are now ready to determine the projectile motion from B to C. Here we have

$$\ddot{x} = 0 \quad \text{and} \quad \ddot{y} = -g. \tag{11}$$

For convenience, we set $t = 0$ to be the time at which A is at B. Hence, using constant acceleration equations, we have that the x and y coordinates of A as a function of time are

$$x = v_{Bx}t \quad \text{and} \quad y = h + v_{By}t - \tfrac{1}{2}gt^2, \tag{12}$$

which account for the fact that $x = 0$ and $y = h$ when $t = 0$, and in which, using Eq. (10),

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$$v_{Bx} = \cos\theta\sqrt{2g(\sin\theta - \mu_k\cos\theta)\ell} \quad \text{and} \quad v_{By} = -\sin\theta\sqrt{2g(\sin\theta - \mu_k\cos\theta)\ell}. \tag{13}$$

Referring to the second of Eqs. (12), and letting t_C be the time when A is at C, we have $y(t_C) = 0$, i.e.,

$$0 = h + v_{By}t_C - \tfrac{1}{2}gt_C^2 \quad \Rightarrow \quad t_C = \frac{1}{g}\left(v_{By} \pm \sqrt{v_{By}^2 + 2gh}\right). \tag{14}$$

Observing that the only physically meaningful root is the one with the plus sign in front of the square root, and substituting this result into the first of Eqs. (12), we have

$$d = \frac{v_{Bx}}{g}\left(v_{By} + \sqrt{v_{By}^2 + 2gh}\right). \tag{15}$$

where we have accounted for the fact that $d = x(t_C)$. Recalling that $g = 32.2\,\text{ft/s}^2$, $\theta = 30°$, $\mu_k = 0.3$, $\ell = 25\,\text{ft}$, and $h = 5\,\text{ft}$, we first evaluate v_{Bx} and v_{By} in Eqs. (13) (this gives $v_{Bx} = 17.03\,\text{ft/s}$ and $v_{By} = -9.832\,\text{ft}$), and then we use these values (to their full precision) in Eq. (15) to obtain

$$\boxed{d = 5.622\,\text{ft}.} \tag{16}$$

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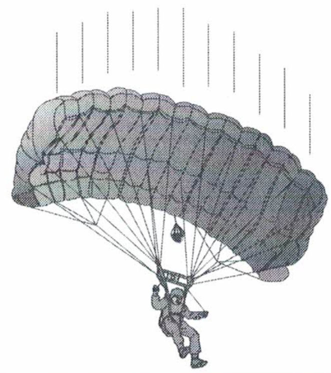
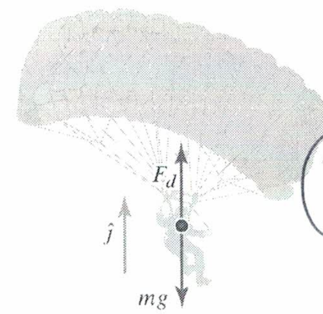
Problem 3.18

As the skydiver moves downward with a speed v, the air drag exerted by the parachute on the skydiver has a magnitude $F_d = C_d v^2$ (C_d is a drag coefficient) and a direction opposite to the direction of motion. Determine the expression of the skydiver's acceleration in terms of C_d, v, the mass of the skydiver m, and the acceleration due to gravity.



Solution

We model the skydiver as a particle moving only in the vertical direction. We assume that the forces acting on the skydiver are only gravity and the air resistance. Hence, the FBD of the skydiver is that shown at the right.



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Balance Principles. Applying Newton's second law in the vertical direction, we have

$$\sum F_y: \quad F_d - mg = ma_y, \tag{1}$$

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where F_d is the air drag force due to the parachute and a_y is vertical component of the skydiver's acceleration.

Force Laws. As indicated in the problem statement, the force law for F_d is

$$F_d = C_d v^2. \tag{2}$$

Kinematic Equations. Since we are solving for the acceleration directly, we do not need any special kinematic equations.

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Computation. Substituting Eq. (2) into Eq. (1) and solving for a_y, we have

$$a_y = \frac{C_d}{m} v^2 - g. \tag{3}$$

Finally, expressing our answer in vector form, we have that the acceleration of the skydiver is

$$\boxed{\vec{a} = \left(\frac{C_d}{m} v^2 - g\right) \hat{\jmath}.}$$

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