Problem 2.1

10

If $\vec{v}_{avg}$ is the average velocity of a point P over a given time interval, is $|\vec{v}_{avg}|$, the magnitude of the average velocity, equal to the average speed of P over the time interval in question?

Binary grading here

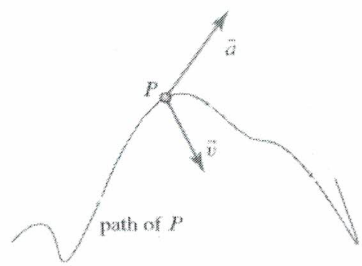
Solution

In general, $|\vec{v}_{avg}|$ is not equal to v_{avg}. To see this, consider a car that drives along a loop of length ΔL over a time interval Δt such that the departure and arrival points coincide. Since the departure and arrival positions coincide, $\vec{v}_{avg}$ is equal to zero. This, implies that $|\vec{v}_{avg}|$ is also equal to zero. By contrast, the average speed will be different from zero because it is equal to the ratio $\Delta L/\Delta t$.

Problem 2.3

10

Is it possible for the vector $\vec{v}$ shown to represent the velocity of the point P?



Solution

No, because the vector $\vec{v}$ shown is not tangent to the path at point P, which it must.

Quick reference:

2.7:

$\boxed{\vec{v}_{avg} = \vec{0}.}$ $\boxed{v_{avg} = 5.556\,\text{m/s}.}$

2.11:

$\boxed{\vec{v}(15\,\text{s}) = (72.08\,\hat{\imath} + 6.240\,\hat{\jmath})\,\text{ft/s}, \quad v(15\,\text{s}) = 72.34\,\text{ft/s}, \quad \vec{a}(15\,\text{s}) = -(15.76\,\hat{\imath} + 1.393\,\hat{\jmath})\,\text{ft/s}^2.}$

2.43:

$\boxed{\beta = 93.75\,\text{ft/s}^{5/2}.}$

2.96:

$\boxed{t_{flight} = 6.263\,\text{s}.}$ $\boxed{\vec{r}_{land} = (281.8\,\hat{\imath} - 4.500\,\hat{\jmath})\,\text{m}.}$

2.138:

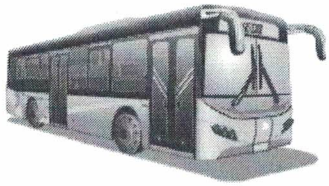
$\boxed{\dot{r}\big|_{\theta=90°} = 8.825\,\text{ft/s} \quad \text{and} \quad \dot{\phi}\big|_{\theta=90°} = -0.7746\,\text{rad/s}.}$

2.132:

$\boxed{\dot{r} = -1.204\,\text{m/s} \quad \text{and} \quad \dot{\theta} = -0.8504\,\text{rad/s}.}$

Problem 2.7

A city bus covers a 15 km route in 45 min. If the initial departure and final arrival points coincide, determine the average velocity and the average speed of the bus over the entire duration of the ride. Express the answers in m/s.



Solution

Since the departure and arrival points coincide, the displacement vector over the duration of the ride is equal to zero. This implies that the average velocity of the bus over the duration of the ride is equal to zero:

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$$\boxed{\vec{v}_{\text{avg}} = \vec{0}.}$$

Letting d denote the total distance traveled by the bus and letting Δt denote the time to travel the distance d, the average speed over the duration of the ride is

$$v_{\text{avg}} = \frac{d}{\Delta t}. \tag{1}$$

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Since $d = 15\,\text{km} = 15\times10^3\,\text{m}$ and $\Delta t = 45\,\text{min} = 2700\,\text{s}$, we can evaluate the above expression to obtain

$$\boxed{v_{\text{avg}} = 5.556\,\text{m/s}.}$$

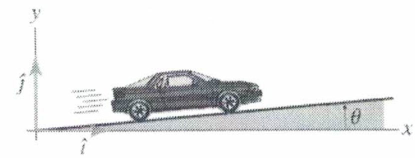
-1 if calculation error

Problem 2.11

The position of a car as a function of time t, with $t > 0$ and expressed in seconds, is

$$\vec{r}(t) = [(5.98t^2 + 0.139t^3 - 0.0149t^4)\,\hat{\imath} + (0.523t^2 + 0.0122t^3 - 0.00131t^4)\,\hat{\jmath}]\ \text{ft}.$$

Determine the velocity, speed, and acceleration of the car for $t = 15$ s.



Solution

The velocity is obtained by taking the derivative of the position with respect to time. This gives

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$$\vec{v} = [(11.96t + 0.4170t^2 - 0.05960t^3)\,\hat{\imath} + (1.046t + 0.03660t^2 - 0.005240t^3)\,\hat{\jmath}]\ \text{ft/s}. \tag{1}$$

The speed is the magnitude of the velocity. Using Eq. (1), we have

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$$v = \sqrt{(11.96t + 0.4170t^2 - 0.05960t^3)^2 + (1.046t + 0.03660t^2 - 0.005240t^3)^2}\ \text{ft/s}, \tag{2}$$

which can be simplified to

$$v = t\sqrt{144.1 + 10.05t - 1.261t^2 - 0.05009t^3 + 0.003580t^4}\ \text{ft/s}. \tag{3}$$

The acceleration is computed by taking the derivative of the velocity with respect to time. Using Eq. (1), we have

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$$\vec{a} = [(11.96 + 0.8340t - 0.1788t^2)\,\hat{\imath} + (1.046 + 0.07320t - 0.01572t^2)\,\hat{\jmath}]\ \text{ft/s}^2. \tag{4}$$

Evaluating Eqs. (1), (3), and (4) for $t = 15$ s, we have

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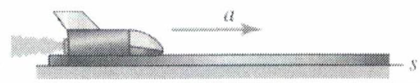
$$\boxed{\vec{v}(15\,\text{s}) = (72.08\,\hat{\imath} + 6.240\,\hat{\jmath})\ \text{ft/s}, \quad v(15\,\text{s}) = 72.34\ \text{ft/s}, \quad \vec{a}(15\,\text{s}) = -(15.76\,\hat{\imath} + 1.393\,\hat{\jmath})\ \text{ft/s}^2.}$$

Problem 2.43

The acceleration of a sled is prescribed to have the following form: $a = \beta\sqrt{t}$, where t is time expressed in seconds, and β is a constant. The sled starts from rest at $t = 0$. Determine β in such a way that the distance traveled after 1 s is 25 ft.



Solution

The acceleration is $a = dv/dt$. Since a is given as a function of time, we can separate the velocity and time variable as follows:

$$\frac{dv}{dt} = a(t) \quad \Rightarrow \quad dv = a(t)\,dt. \tag{1}$$

3

Recalling that $a(t) = \beta\sqrt{t}$ and that the velocity is equal to zero for $t = 0$, we integrate the last of Eqs. (1) as follows:

$$\int_0^v dv = \int_0^t \beta\sqrt{t}\,dt \quad \Rightarrow \quad v(t) = \tfrac{2}{3}\beta t^{3/2}. \tag{2}$$

We recall that the velocity is ds/dt, so that we can write

$$\frac{ds}{dt} = v(t) \quad \Rightarrow \quad ds = v(t)\,dt. \tag{3}$$

Letting s_0 denote the value of s for $t = 0$ and using the expression for $v(t)$ in the last of Eqs. (2), we can integrate the last of Eqs. (3):

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$$\int_{s_0}^s ds = \int_0^t \tfrac{2}{3}\beta t^{3/2}\,dt \quad \Rightarrow \quad s - s_0 = \tfrac{4}{15}\beta t^{5/2}. \tag{4}$$

Letting d denote the distance traveled during $\Delta t = 1$ s, we can rewrite the last of Eqs. (4) as follows:

1.5

$$d = \tfrac{4}{15}\beta \Delta t^{5/2}, \tag{5}$$

which can be solved for β to obtain

1.5

$$\beta = \frac{15d}{4\Delta t^{5/2}}. \tag{6}$$

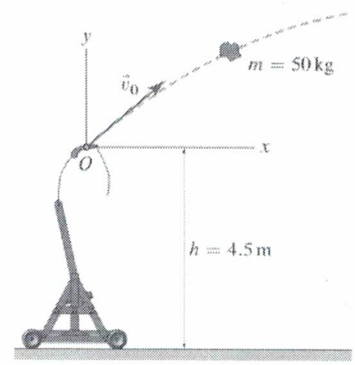
Recalling that $d = 25$ ft and that $\Delta t = 1$ s, we can evaluate Eq. (6) to obtain

1

$$\boxed{\beta = 93.75\ \text{ft/s}^{5/2}.}$$

Problem 2.96 ǃ

A trebuchet releases a rock with mass $m = 50\,\text{kg}$ at point O. The initial velocity of the projectile is $\vec{v}_0 = (45\,\hat{\imath} + 30\,\hat{\jmath})\,\text{m/s}$. Neglecting aerodynamic effects, determine where the rock will land and its time of flight.



Solution

Referring to the coordinate system defined in the problem statement, we see that y_{land}, the y coordinate of the rock when it lands on the ground, is $-h$. With this in mind, we can write the following constant acceleration equation for the y coordinate of the rock:

$$y = v_{0y}t - \tfrac{1}{2}gt^2,$$

where it is understood that $t = 0$ is the time of release and $y = 0$ and v_{0y} are the vertical position and the vertical component of velocity of the rock at time $t = 0$, respectively. Denoting by t_{flight} the time at which the rock impacts the ground, we have

$$-h = v_{0y}t_{\text{flight}} - \tfrac{1}{2}gt_{\text{flight}}^2 \quad \Rightarrow \quad gt_{\text{flight}}^2 - 2v_{0y}t_{\text{flight}} - 2h = 0 \quad \Rightarrow \quad t_{\text{flight}} = \frac{v_{0y} \pm \sqrt{v_{0y}^2 + 2gh}}{g}. \tag{1}$$

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The only physically meaningful solution for t_{flight} is that corresponding to the $+$ sign in front of the square root, that is,

$$t_{\text{flight}} = \frac{1}{g}\left(v_{y0} + \sqrt{v_{y0}^2 + 2gh}\right). \tag{2}$$

Recalling that $g = 9.81\,\text{m/s}^2$, $v_{0y} = 30\,\text{m/s}$, and $h = 4.5\,\text{m}$, we can evaluate the expression above to obtain

$$\boxed{t_{\text{flight}} = 6.263\,\text{s}.}$$

1

Next observing that the motion is in the x direction is a constant acceleration motion with acceleration equal to zero, the x coordinate of the rock is described by the following (constant acceleration) equation:

$$x = v_{0x}t, \tag{3}$$

where we have accounted for the fact that $x = 0$ for $t = 0$, and where v_{0x} is the x component of the velocity of the rock for $t = 0$. Substituting Eq. (2) into Eq. (3), for $t = t_{\text{flight}}$ we have

4

$$x_{\text{land}} = \frac{v_{0x}}{g}\left(v_{y0} + \sqrt{v_{y0}^2 + 2gh}\right). \tag{4}$$

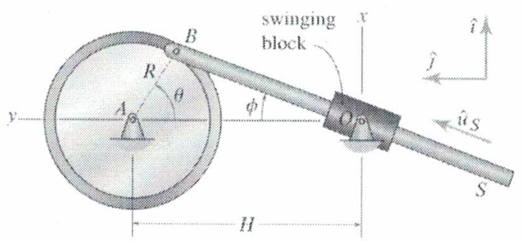
The position of the rock when the rock hits the ground is $\vec{r}_{\text{land}} = x_{\text{land}}\,\hat{\imath} - h\,\hat{\jmath}$. Therefore, recalling that $g = 9.81\,\text{m/s}^2$, $v_{0x} = 45\,\text{m/s}$, $v_{0y} = 30\,\text{m/s}$, and $h = 4.5\,\text{m}$, and using Eq. (4) to evaluate x_{land} we have

$$\boxed{\vec{r}_{\text{land}} = (281.8\,\hat{\imath} - 4.500\,\hat{\jmath})\,\text{m}.}$$

2

Problem 2.138 I

The mechanism shown is called a *swinging block* slider crank. First used in various steam locomotive engines in the 1800s, this mechanism is often found in door-closing systems. If the disk is rotating with a constant angular velocity $\dot\theta = 60\,\text{rpm}$, $H = 4\,\text{ft}$, $R = 1.5\,\text{ft}$, and r is the distance between B and O, compute $\dot r$ and $\dot\phi$ when $\theta = 90°$. *Hint:* Apply Eq. (2.48) to the vector describing the position of B relative to O.



Solution

We can express the velocity of B in two ways. First, as the time derivative of the position vector $\vec r_{B/A}$ and second as the time derivative of the position vector $\vec r_{B/O}$. Referring to the figure in the problem statement, we can express these two position vectors as follows:

2

$$\vec r_{B/A} = R\,\hat u_{B/A} \quad\text{and}\quad \vec r_{B/O} = r\,\hat u_S, \tag{1}$$

where, as given the problem statement, r is the distance between B and O, and where we observe that the angular velocities of the unit vector in the above equations are

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$$\vec\omega_{\hat u_{B/A}} = \dot\theta\,\hat k \quad\text{and}\quad \vec\omega_{\hat u_S} = -\dot\phi\,\hat k. \tag{2}$$

2

Hence, observing that $\dot R = 0$ since R is a constant, using Eq. (2.48) on p. 81 of the textbook, we have

$$\vec v_B = \dot{\vec r}_{B/A} = \dot\theta\,\hat k \times R\,\hat u_{B/A} \quad\text{and}\quad \vec v_B = \dot{\vec r}_{B/O} = \dot r\,\hat u_S - \dot\phi\,\hat k \times r\,\hat u_S. \tag{3}$$

Next, we observe that, for $\theta = 90°$, we have

4

$$\hat u_{B/A} = \hat\imath, \quad r = \sqrt{H^2 + R^2}, \quad\text{and}\quad \hat u_S = \frac{1}{\sqrt{R^2+H^2}}(R\,\hat\imath + H\,\hat\jmath). \tag{4}$$

Substituting Eqs. (4) into Eqs. (3), for $\theta = 90°$, we have

$$\vec v_B\big|_{\theta=90°} = R\dot\theta\,\hat\jmath \quad\text{and}\quad \vec v_B\big|_{\theta=90°} = \frac{\dot r}{\sqrt{R^2+H^2}}(R\,\hat\imath + H\,\hat\jmath) + H\dot\phi\,\hat\imath - R\dot\phi\,\hat\jmath. \tag{5}$$

Equating the two above expressions for $\vec v_B$ component by component, we have

$$\hat\imath:\quad \frac{\dot r R}{\sqrt{R^2+H^2}} + H\dot\phi = 0, \tag{6}$$

$$\hat\jmath:\quad \frac{\dot r H}{\sqrt{R^2+H^2}} - R\dot\phi = R\dot\theta. \tag{7}$$

4

Equations (6) and (7) form a system of two equations in the two unknowns $\dot r$ and $\dot\phi$ (at $\theta = 90°$) whose solution is

$$\dot r\big|_{\theta=90°} = \frac{RH\dot\theta}{\sqrt{R^2+H^2}} \quad\text{and}\quad \dot\phi\big|_{\theta=90°} = -\frac{R^2\dot\theta}{R^2+H^2}. \tag{8}$$

2

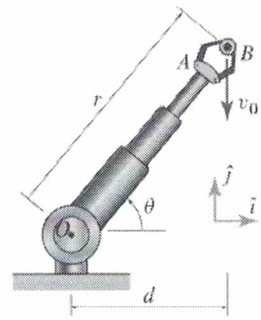
Recalling that we have $\dot\theta = 60\,\text{rpm} = 60(2\pi/60)\,\text{rad/s}$, $H = 4\,\text{ft}$, $R = 1.5\,\text{ft}$, we can evaluate the quantities in Eqs. (8) to obtain

$$\boxed{\dot r\big|_{\theta=90°} = 8.825\,\text{ft/s} \quad\text{and}\quad \dot\phi\big|_{\theta=90°} = -0.7746\,\text{rad/s}.}$$

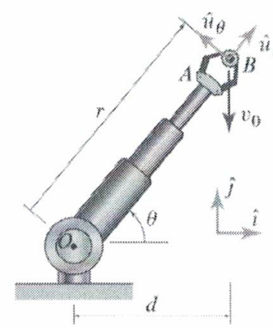
1

Problem 2.132

The end B of a robot arm is moving vertically down with a constant speed $v_0 = 2\,\mathrm{m/s}$. Letting $d = 1.5\,\mathrm{m}$, apply Eq. (2.48) to determine the rate at which r and θ are changing when $\theta = 37°$.



Solution



Referring to the figure on the right, we begin by describing the position of point B relative to O using the $(\hat{u}_r, \hat{u}_\theta)$ component system:

$$\vec{r}_B = r\,\hat{u}_r. \tag{1}$$

The velocity of B is $\vec{v}_B = \dot{\vec{r}}_B$. Then, using Eq. (2.48) on p. 81 of the textbook, we have

3

$$\vec{v}_B = \dot{r}\,\hat{u}_r + \vec{\omega}_r \times r\,\hat{u}_r, \tag{2}$$

where $\vec{\omega}_r$ is the angular velocity of the vector $\vec{r}_B$. Since the vector $\vec{r}_B$ rotates with the robotic arm, we have

$$\vec{\omega}_r = \dot{\theta}\,\hat{k}. \tag{3}$$

Substituting Eq. (3) into Eq. (2) we have

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$$\vec{v}_B = \dot{r}\,\hat{u}_r + \dot{\theta}\,\hat{k} \times r\,\hat{u}_r \quad \Rightarrow \quad \vec{v}_B = \dot{r}\,\hat{u}_r + \dot{\theta} r\,\hat{u}_\theta. \tag{4}$$

Since point B is moving downward along a vertical line with speed v_0, using the $(\hat{\imath}, \hat{\jmath})$ component system, the velocity of B can also be described as follows:

$$\vec{v}_B = -v_0\,\hat{\jmath}. \tag{5}$$

3

We now observe that

$$\hat{\jmath} = \sin\theta\,\hat{u}_r + \cos\theta\,\hat{u}_\theta. \tag{6}$$

Therefore, Eq. (5) can be rewritten as

$$\vec{v}_B = -v_0(\sin\theta\,\hat{u}_r + \cos\theta\,\hat{u}_\theta). \tag{7}$$

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Equating the second of Eqs. (4) and Eq. (5) component by component, we have

$$\dot{r} = -v_0 \sin\theta \quad \text{and} \quad \dot{\theta} r = -v_0 \cos\theta. \tag{8}$$

Recognizing that $r\cos\theta = d$, i.e., $r = d/\cos\theta$, we can solve Eqs. (8) for $\dot{r}$ and $\dot{\theta}$ to obtain

$$\dot{r} = -v_0 \sin\theta \quad \text{and} \quad \dot{\theta} = -\frac{v_0 \cos^2\theta}{d}. \tag{9}$$

4

Finally, recalling that $v_0 = 2\,\mathrm{m/s}$, $\theta = 37°$, and $d = 1.5\,\mathrm{m}$, Eqs. (9) can be evaluated to obtain

$$\boxed{\dot{r} = -1.204\,\mathrm{m/s} \quad \text{and} \quad \dot{\theta} = -0.8504\,\mathrm{rad/s}.}$$

1